Dr. P. G. Klemens.
Physics.

PROCEEDINGS OF CONFERENCE ON AUTOMATIC COMPUTING MACHINES

held in the Department of Electrical Engineering, University of Sydney

August 1951

COMMONWEALTH SCIENTIFIC AND INDUSTRIAL RESEARCH ORGANIZATION IN CONJUNCTION WITH THE DEPARTMENT OF ELECTRICAL ENGINEERING OF THE UNIVERSITY OF SYDNEY

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CONFERENCE ON AUTOMATIC COMPUTING MACHINES

SYDNEY, 7th - 9th AUGUST, 1951

ADDRESS OF WELCOME

BY EMERITUS PROFESSOR SIR JOHN MADSEN, Kt., D.Sc., B.E.

I have pleasure in offering you a cordial welcome to the Conference on Automatic Computing Machines.

In view of the attention which has been given to this subject in all parts of the world in the last decade, it is very gratifying to see that such a large gathering, and one representing such a wide variety of interests, has found it possible to attend the first general conference to be held in Australia in this rapidly expanding subject.

While recognising and appreciating very highly the great honour which has been done to me in asking me to take the chair at the opening session of this Conference, I desire to express my indebtedness to my colleague, the Professor of Electrical Engineering, D. M. Myers, who has taken a keen interest in this field of work since his return from Great Britain in 1936. Besides initiating the action which has led to C.S.I.R.O. setting up aids to mathematical computing, and ultimately to the decision to hold this Conference, I am indebted to him for the information he has given me regarding the general objectives of the Conference and of the enterprises with which it will deal.

Computing has traditionally occupied an intermediate position between theory and practice, and being neither fish nor fowl, has been looked at rather askance in the past by both species. But modern science and technology, with their rapid expansion accelerated by the demands of two world wars, have placed a correspondingly increasing requirement on the computer; consequently, both the producer and the user of mathematical theory have closed their ranks in stimulating the progress of computing methods and techniques. The resulting progress can only be described as spectacular; so much so that its protagonists are at considerable pains to keep abreast of it.

It is for these reasons that this Conference has been convened. The Conference is to take place in two sessions, the first of which is arranged primarily for the "user"; it is hoped that this session will serve as a clearing house for examining facilities in the light of requirements. Attention will naturally be given to overseas developments as an indication of what the future holds. However, the nature of this first session demands also an examination of facilities available in this country and of the way in which they may be applied to meeting Australia's relevant needs. The second session is intended primarily for those who are directly concerned in computing and with computing devices.

In tracing the development of computing machines, one feels impelled to pay tribute to the contribution of the business machine manufacturers; their inspiration and urge came, in the first instance, from commerce, and one of their major objectives was to achieve higher and higher speeds of computation. This was an obvious economic requirement of the large business houses in which calculations, admittedly simple as mathematical operations, had to be carried out in such vast numbers that they occupied an important place in the activities of the organization concerned. It is instructive to observe how the quest for higher speeds has influenced the modern treatment of problems of a more scientific nature. This aspect of computing will receive considerable attention during the present Conference;

let it suffice for the time being to acknowledge the extent to which the now thoroughly established types of computing machine, designed to meet commercial needs, have provided a sound foundation for the more recent developments aimed at meeting the demands of productive industry and scientific research. Much of the initiative in using standard commercial machines for scientific calculations came from the late Dr. L. J. Comrie, F.R.S.

Before the last war, a great deal of progress was made in the class of machine known as "analogue", radically different from the conventional class of "digital" machine about which I have just spoken. These developments took place mainly in the U.S.A. and Great Britain, but the second world war provided an urgent incentive which led to rapid progress in most parts of the world. Analogue machines already in existence were put to extensive use, and new machines, either for general application or for specific military purposes, sprang into being. It is probable that many of the more advanced of these devices are still known to relatively few people because of the belligerent purposes they serve. However, the war-time activities in this field have provided many new methods and devices of importance in civil life.

Meanwhile, steady progress was maintained in the field of digital machines, the main trends being towards greater flexibility, greater capacity and increased use of automatic and semi-automatic handling of computing routines, notably through the widening application of punched-card and punched-tape methods, together with the facilities offered by these methods for automatic sorting and tabulation.

All these advances resulted in a growing appreciation by industries, and by those engaged in physical and other sciences, of the assistance that could be derived from modern machines in bridging the gap between mathematical theory and practical application. But the last decade has brought such a spectacular achievement in the modern high-speed automatic digital calculating machine, that mathematical computing can be said to have entered an entirely new phase. Again, the major developments have occurred in the U.S.A. and Great Britain.

During the short period of this Conference, a review will be made of the progress in world-wide development of automatic computing machines. We are fortunate in having with us as an honoured guest, Professor D. R. Hartree, F.R.S., Plummer Professor of Mathematical Physics in the University of Cambridge. Professor Hartree has come to Australia on the invitation of C.S.I.R.O. and of the Australian National University, and has expressed his willingness to lead some of the discussions.

Professor Hartree, whilst occupying the Chair of Applied Mathematics at the University of Manchester, became interested in the mechanical integration of differential equations, and after making a study of the differential analyser developed by Dr. Bush, at the Massachusetts Institute of Technology, was responsible for the establishment in 1935 of the first differential analyser to be built in Great Britain. He guided the application of this machine to a wide range of industrial and scientific problems and his advice was sought in respect to new machines in Great Britain and even in Australia.

Professor Hartree has also taken a most active part in the development of high-speed automatic digital machines, and has travelled extensively in the U.S.A. and Europe in following this interest. You will agree that our forthcoming discussion will take place against a background of world progress which Professor Hartree is singularly well qualified to describe.

And this brings us to the Australian scene. We are already well provided in this country with computing devices of the conventional pattern, which have been extensively applied to scientific problems, particularly in the various fields of applied statistics. Apart from Government and other establishments which have set up facilities for specific purposes, there is also, within C.S.I.R.O.,a well-equipped Section of Mathematical Statistics, which serves the rather more general purpose of participating in co-operative research with experimental laboratories, mainly in the agriculatural sciences. The resources of this Section have been used in a very wide field of application. Similarly, the resources of various government and private establishments - such as the government statisticians - have been readily made available from time to time for scientific work.

The development of analogue and digital machines of the most recent automatic types has taken place mainly within C.S.I.R.O. A Section of Mathematical Instruments was established several years ago within the C.S.I.R.O. Division of Electrotechnology, and now has a separate existence within this Department of Electrical Engineering of the University of Sydney. This Section has developed a differential analyser which is now fully operative and has been applied to rather a wide range of scientific and industrial problems. The Section is also concerned in high-speed digital computing, and some of this work will be discussed during the Conference.

The C.S.I.R.O. Division of Radiophysics, situated in the grounds of this University, has installed a Hollerith machine for carrying out scientific work, and is engaged in the construction of a high-speed electronic digital given fixed limits (as distinct from an integral as a function of its upper limit), a differential analyser to the integration of differential equations; I know of no general-purpose instrument. Also the accuracy of any instrument is limited by the accuracy of the mechanical and electrical components of which it is constructed, and by the attainable accuracy of physical measurement. However, within their limitations, instruments are very valuable aids to calculation.

Examples of calculating machines are the standard desk machines such as the Brunsviga and Marchant, punched-card equipment such as the IBM Tabulator and multiplying punch, and the automatic general-purpose machines considered later (§3). Since they work with numbers in digital form, they can only be applied to calculations which can be reduced to finite sequences of arithmetical operations, and in particular they cannot deal continuously with continuous variables. But they can be designed to be used for any calculation which can be so reduced; further their accuracy is not limited to the accuracy of the physical components of which they are constructed, and they can in principle be used for calculations of any finite degree of accuracy within their capacity.

2. The differential analyser.

An outstanding example of a mathematical instrument is the differential analyser, an instrument for obtaining solutions of differential equations by mechanical means. A "differential equation" is the formal expression of a relation between the rate of change of a variable quantity and the magnitude of that quantity itself. Consider, for example, the motion of a projectile through the air. Its acceleration, which is the rate of change of its velocity, depends on the resistance of the air, which itself depends on the velocity; that is, the rate of change of velocity depends in a definite way on the velocity itself. The expression of this relation is a differential equation.

Such equations arise in the quantitative treatment of problems in a wide range of scientific and technical subjects, for example vibrations and stability of mechanical and electrical systems of various kinds, chemical kinetics, fluid dynamics, meteorology, and the structures of atoms and of stars. Sometimes formal solutions in finite terms can be found by standard methods, but often this is not possible; however, in very many cases quantitative numerical information about the solution is wanted. Thus there is an important practical requirement to obtain numerical solutions of differential equations, irrespective of whether they have formal solutions in terms of tabulated functions. The differential analyser provides a means of doing this for a wide range of ordinary differential equations, and its use can be extended to the approximate solution of a more limited range of partial differential equations (7, 20).

In solving a differential equation, the basic process is that of integration, which is carried out in the differential analyser by means of a continuously variable gear. If the gear has a gear ratio 1:n between the rotations of the input and output shafts, and is of such a mechanical construction that n can be varied while the input shaft is rotating, then for a rotation x turns of the input shaft, the rotation of the output shaft is $\int n\,dx$ turns (4, 20). This principle has been realised practically in several forms (9, 20, 46).

The differential analyser consists of an assembly of units for carrying out the various processes which may be involved in the integration of a differential equation. The units are interconnected by shafts, or by other devices replacing them, each of which represents by its rotation one of the quantities involved in the equation, and provides a measure of that quantity. The analyser is set up in such a way that the relative rotations of the various shafts are related in the way expressed by the differential equation to be solved.

3. Automatic general-purpose digital machines.

The main developments of calculating equipment in the last ten or twelve years have been in digital machines with two features, which are expressed by calling them automatic and general-purpose or universal. "Automatic" means that once the machine has been supplied with a specification, in suitable form, of the calculation to be carried out, it proceeds with the work without further attention on the part of the operator. This specification of the calculation may take various forms; it may, for example, consist of a set of connections made on a plugboard and the settings of a group of switches, or a set of punchings on cards or on tape; in any case it may be considered as consisting of a set of operating instructions to be taken in a definite order.

"General-purpose" or "universal" means that the same machine can be applied to a wide range of calculations of different kinds by providing it with the appropriate schedules of operating instructions, so that one and the same machine can be used, for example, for evaluating solutions of sets of linear simultaneous algebraic equations, for step-by-step numerical solution of ordinary differential equations, for summation of series, for constructing tables of prime numbers, and for many other kinds of numerical work.

The general idea of such a machine is not new. It is over a hundred years old and is due to Charles Babbage, whose projected "analytical engine" was to be a general-purpose digital machine, automatic except in one feature, concerned with the use of tables of functions. But it is only relatively recently that Babbage's ideas have come to be realised, and then in a physical form much different from anything he could have foreseen.

The first automatic general-purpose digital machine to be completed is the Automatic Sequence-controlled Calculator (Harvard Mark I Calculator) at Harvard University; this was planned before the war, but only completed during it. This machine in its original form has been fully described (21) and an account has also been published of various later developments (22). The first machine using electronic circuits, with the speed of operation they make possible, was the ENIAC (Electronic Numerical Integrator And Calculator) built at the University of Pennsylvania for the U.S. Army Ordnance Laboratories at Aberdeen Proving Ground, Maryland. Accounts have also been published of this machine. (3, 12, 19, 39).

Any automatic digital calculating machine must comprise a store for numerical data, intermediate results, and operating instructions, an arithmetical unit for carrying out arithmetical operations on numbers transferred to it from the store, means of transfer of numbers between the store and arithmetical unit, and a control system to organise these transfers and the operations carried out by the arithmetical unit on the numbers transferred to it.

It must also have an input unit for accepting from the outside world numerical data and operating instructions for the calculation, and an output unit for furnishing to the outside world the results of its work. These units may not be all physically distinct; for example, in the Harvard Mark I Machine, and in the ENIAC, the registers which form the store are also adding units and so form part of the arithmetical unit. And the store may not all be of the same physical kind; for example there may be one store for numbers and another of a different kind for instructions; or it may be divided into a main store with direct access to the arithmetical unit and to the control unit, and an auxiliary store into which numbers or instructions which will not be wanted for some time can be transferred from the main store (see fig. 1).

In both the Harvard Mark I machine and the ENIAC, and in others of the early machines, numbers and operating instructions are represented within the machine in quite different forms; for example in the Harvard Mark I machine numbers are represented by the positions of sets of counting wheels, whereas instructions are represented by punchings on a paper tape, and in the ENIAC numbers are represented by the states of sets of electronic valves connected to form counting rings, whereas operating instructions are represented by switch settings and by interconnections of its various units.

4. Recent developments in digital machines.

Of the more recent developments, the one which is most important in principle is the use of the same form, in the machine, for operating instructions as for numbers. This has two consequences; one is that the same store can be used for numbers and operating instructions, and secondly, and

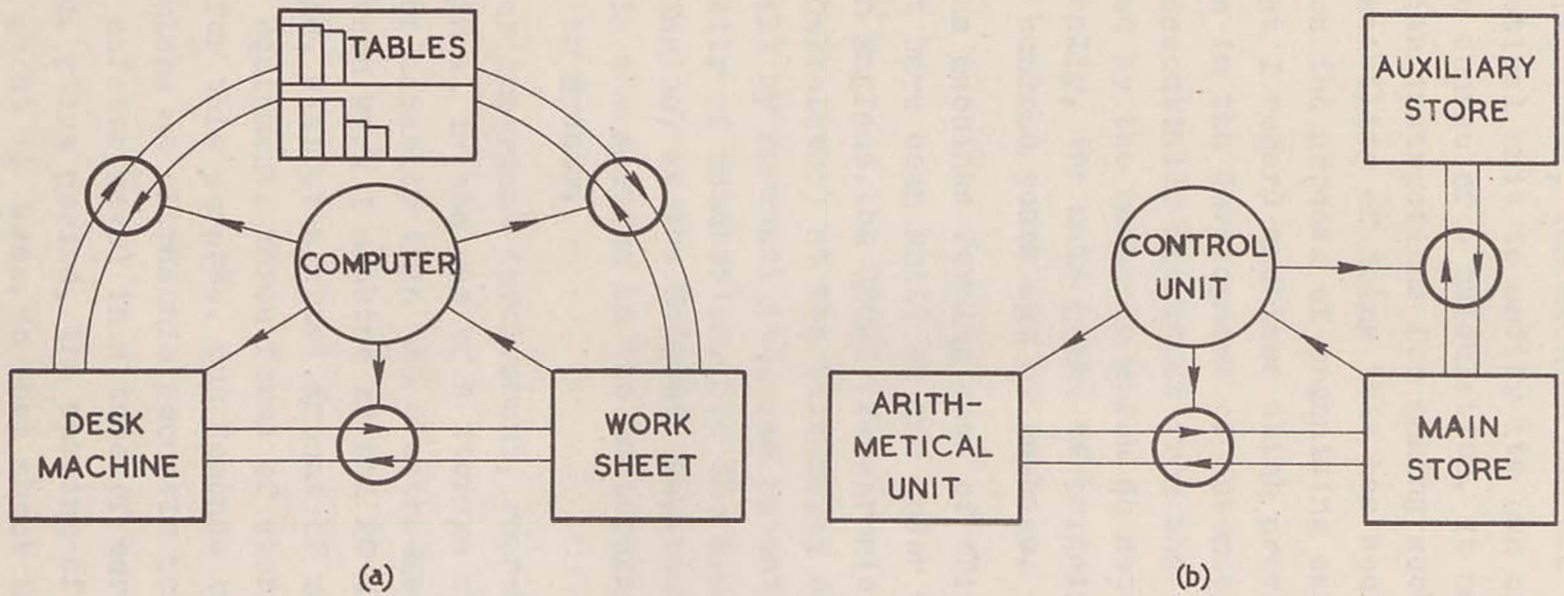

FIG. I.

a) ORGANIZATION OF A HAND CALCULATION CARRIED OUT WITH AID OF A DESK MACHINE.

b) ORGANIZATION OF A CALCULATION WITH AN AUTOMATIC DIGITAL MACHINE.

much more important, the machine can use the facilities of the arithmetical unit to modify its own operating instructions in the course of a calculation. It must of course be provided with instructions for making such modifications, and the possibility of doing this has such a profound influence on the process of organising calculations for the machine that I regard machines which provide it as forming a new stage in the development of automatic calculating machines, essentially different from the first stage, now past, formed by the machines which do not provide it. It is incidentally, the main point of principle which was not foreseen a hundred years ago by Babbage.

Various machines forming part of this second stage of development have been built or are under construction, including, in England, the EDSAC (Electronic Delay Storage Automatic Calculator) at the University of Cambridge, the machine built by Ferranti Ltd. and recently installed in the University of Manchester, and the A.C.E. (Automatic Computing Engine) at the National Physical Laboratory, and in Australia the machine in the Radiophysics Division of C.S.I.R.O. in Sydney.

Another important development, represented in all these machines, is the use of a storage system providing much greater capacity than the electro-mechanical counters of the Harvard Mark I machine or the 20 electronic counters of the ENIAC, without a great amount of mechanical and electrical equipment. Three forms of storage have been developed for this purpose. One depends on the storage of a train of pulses as ultrasonic acoustic pulses (in most cases on a carrier wave) in a tube of mercury (43, 44, 55); at 1 μ sec. pulse period, the spacing of the pulses in the mercury is about $1\frac{1}{2}$ μsec. so that about 1.000 pulses can be stored in a length of $1\frac{1}{2}$ metres of mercury, and this is enough to represent about 30 numbers to an accuracy of ten decimal digits each; only a small amount of electronic equipment is required for the circulation of the pulses and for providing gates for reading them into and out of the mercury column. This system is used in the EDSAC and ACE, and in the C.S.I.R.O. machine, among others.

Another kind of storage is based on the use of a pattern of electric charge on an insulating screen, in practice the screen of a cathode ray tube. One storage system of this kind, using standard cathode ray tubes, has been developed by Professor F. C. Williams (58) at the University of Manchester, and is used in the Ferranti machine recently installed there; others, involving the use of special cathode-ray tubes, are being developed in the U.S.A.

A third form of storage is based on the distribution of magnetisation of a magnetic material, which may be in the form of a wire, strip, or a film on the curved surface of a cylinder ("magnetic drum"). This magnetic method of storage (see, for example, ref.30) is usually used for the auxiliary store, though in the Harvard Mark III machine it is used for the whole store. Punched cards or punched tape can also be used for auxiliary storage, but use of them usually involves some handling by an operator, so that the process of carrying out the calculation is no longer fully automatic.

II. The C.S.I.R.O. Differential Analyser

by D. M. Myers and W. R. Blunden

1. Introduction.

Professor Hartree (I) has given a brief outline of the principles employed in the differential analyser. A great deal of literature * is available on this instrument in its various forms, mainly in the U.S.A. and Great Britain, and considerable studies have been made of its method of application to the integration of differential equations. The idea of interconnecting a number of integrators in order to find a solution of a differential equation is due to Lord Kelvin (47, 48, 49).

The only major instrument of this type so far completed in Australia was constructed under the guidance of the Mathematical Instruments Section of C.S.I.R.O. and we propose to give a brief description of its design and to demonstrate its application to the solution of a specific problem.

The instrument differs from the early types of differential analyser ø mainly in the method of interconnecting its components. These components include ten integrators, six adding units, six gear boxes, and four plotting tables, together with several auxiliary units. Their interconnection is carried out by an electromagnetic system of transmitters and motors, which replace the mechanical shafts and gears of the earlier instruments. The interconnections are made at a central control unit

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* 4, 5, 17, 18, 34, 32, 40, 31, 14

ø See, for example, refs. 4, 17

by a set of plugs and sockets. This device allows unit construction to be adopted and results in a high degree of flexibility and mobility.

2. Integrators.

As Professor Hartree has pointed out, integration may be carried out by a continuously variable gear. A form of gear in fairly common use for the purpose consists of a flat horizontal disc, rotating about a vertical axis; in contact with it is a small vertical wheel which is free to rotate about a horizontal axis co-planar with that of the disc. When the disc rotates, the wheel is caused also to rotate, due to friction at the point of contact, and if no slip occurs between the wheel and the disc, then the ratio of the angular velocity of the wheel to that of the disc is proportional to the displacement of the point of contact from the centre of the disc. This distance is determined by a lead-screw which is arranged so as to move the disc laterally with respect to the wheel. Considering the shaft of the disc as the input, and that of the wheel as the output, the mechanism is a continuously variable gear whose ratio may be set or varied by means of the lead-screw. It can therefore be used as an integrator, and it should be noted that each of the three variables - argument, integrand and integral - is represented in the form of a rotation of a shaft.

In the conventional design, the wheel rests lightly on the disc, so that the radial width of its contact with the disc is very small compared with the radius of the disc, and the displacement of the point of contact is determinate to a close approximation. On the other hand, the output torque is small and the use of an integrator of this design requires a form of servo-mechanism (see ref. 4) to enable its output shaft to drive other mechanical components without causing slip at the point of contact.

The introduction of the servo-mechanism, often known in this context as a "torque amplifier", bridged the gap between Kelvin's theoretical concept and Bush's first realization of an effective instrument. The torque amplifier nevertheless introduces practical difficulties, particularly in regard to maintenance. In later instruments, the torque amplifier has sometimes been replaced by a servo-mechanism using electrical or optical phenomena as its basis. Devices such as these are usually satisfactory in operation but require a considerable quantity of auxiliary electronic equipment.

The replacement of the servo-mechanism by an electro-magnetic system of direct interconnection in the C.S.I.R.O. instrument affects the load on the output of the integrator. The transmitting unit of the system (to be described later) requires a driving torque which would be sufficient to cause slip in an integrator of the early type, and it was decided to use a type of integrator in common use in such equipments as fire control directors. The integrating wheel of the conventional integrator is replaced by a cylinder with its axis perpendicular to that of the disc and co-planar with it, with two hardened steel balls lying between the cylinder and disc, as shown in fig. 1, which includes also an illustration of the conventional type of integrator. The balls are contained in a cage which can be displaced laterally by a lead-screw, and the whole assembly is compressed together by means of a helical spring.

In this case, the normal force is considerable at the point of contact, which must therefore be considered as an area of contact, and we had some misgivings at first regardin the accuracy of integration. Very extensive tests over a long period proved reassuring and the design was accepted. Further tests, carried out after some months of running, showed that the total errors due to this cause and to slip, in the ten integrators, expressed as an error in the displacement (integrand), were less than 0.1% of the maximal displacement.

Each integrator includes two gear-boxes, one in the drive leading to the lead-screw and the other in the output. These gear boxes serve as multiplying units for the integran and integral respectively; the former provides for simple reduction ratios such as 1:2, 1:5 or 1:20 to be introduced, so that a suitable scale factor may be applied to the displacement of the ball-cage. The output gear-box provides for the introduction of any desired ratio within reasonable limits, and will be described in more detail later.

3. Adding units.

Addition is carried out by means of differential gears the only departure from conventional practice being in deta of mechanical design.

4. Gear boxes.

There are six independent gear-boxes of a similar type to those in the output of each integrator. Their design is based on a device frequently used in such applications as the automatic feed of screw-cutting lathes. Each gear-box contains a double train of spur gears, the intermediate shaft being supported on an adjustable mounting so that it may be clamped in any desired position. A book of tables is available, setting out the gears necessary to achieve over

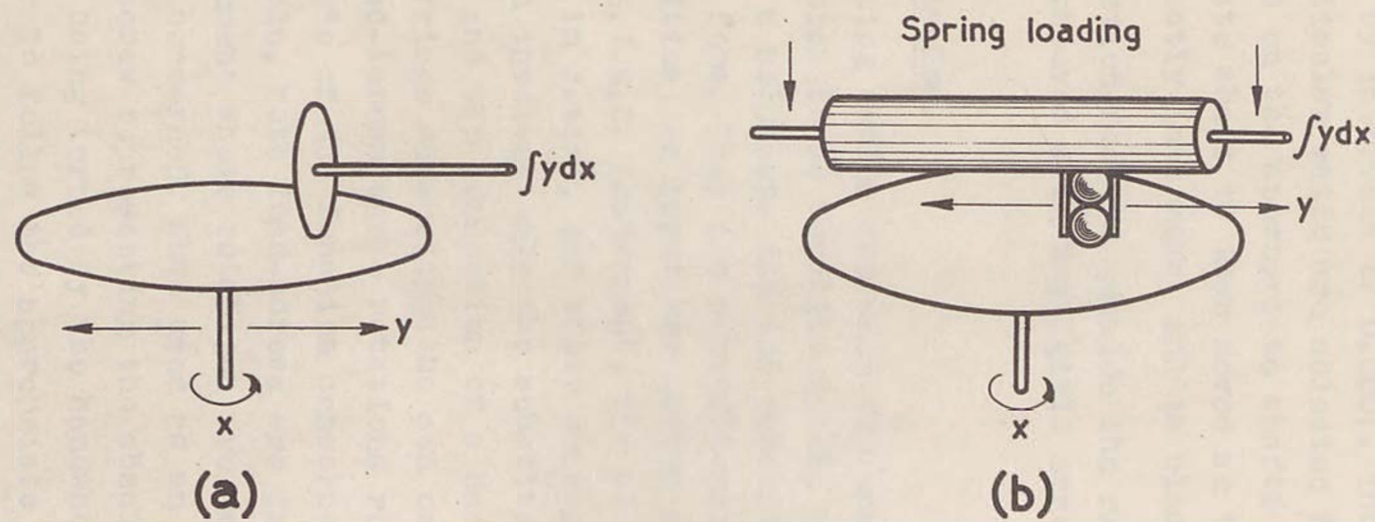

FIG. I. TYPES OF MECHANICAL INTEGRATOR.

a) "KELVIN" WHEEL TYPE.

b) DISC BALLS & CYLINDER TYPE.

reduction factors lying within the range 0.3 to 1, and differing by intervals of 0.0001. The spur gears required for a particular ratio are selected from a gear "library" and placed on the appropriate shafts of the gear-box. The intermediate shaft is then moved so that the gears all come correctly into mesh, and is clamped in position.

Reduction factors outside the range 0.3 to 1 can also be achieved with negligible error.

5. Plotting tables.

Plotting tables are used for enabling functional relationships to be supplied to the instrument during the course of a solution, and for recording the solution in graphical form. They are conventionally known, in those two capacities, as input and output tables respectively. In the C.S.I.R.O. instrument, the plotting tables are identical in design, and their conversion from output to input form involves only the substitution of a cursor for a pen and the connection of a handwheel. The position of the carriage supporting the pen or cursor is controlled by two lead-screws whose rotations represent the abscissa and ordinate of the function concerned. When used as an output table, both lead-screws are driven by elements in the instrument whose rotations are measures of the variables concerned. When used as an input table, only the lead screw representing the abscissa is so driven, the other being turned by the handwheel so as to cause the cursor to follow the appropriate curve. The handwheel simultaneously drives an element in the machine, its rotation thus being made to represent the value of the variable corresponding to the ordinate of the curve.

6. Independent variable drive.

In order to set the instrument in motion, it is necessary to provide an element to represent the independent variable of the equation, and to cause it to rotate at a suitable speed. This is done by means of a displacement-velocity servo-mechanism; this system achieves stable operation for a very wide range of motor speeds.

7. Interconnection of units.

The device used for interconnecting the various components of the instrument is an electromagnetic system of data transmission commonly used by the Admiralty, known as M-type transmission, in which the rotation of the transmitting element is copied at a remote point by a receiving element. The transmitting element takes the form of a commutator having five electrical leads, two of which are connected to a battery or other D.C. source. The three remaining leads are connected to the receiving element, which is in the form of a motor of special design. As the commutator rotates, the two battery leads are connected through it to either two or three of the remaining leads in a repetitive sequence. These three leads go to the three windings on the stator of the receiving motor, and the flow of current in them establishes a magnetic field in the moto The direction of this field is determined by the connections between the leads and the battery. The sequence of these connections established through the transmitting commutator, is such that as the commutator rotates, the magnetic field in the motor rotates, in discrete steps, so as to copy the rotation of the commutator, within limits of angular error determined by the magnitude of the steps (normally 15°). A permanent magnet, forming the rotor of the receiving motor, follows the magnetic field.

The error due to the step-by-step action is made negligible by a suitable choice of scale factors. The motor provides adequate torque for all requirements of the instrument, and the interconnection of units requires only the plugging-in of a three-wire cable; reversal of direction is very easily achieved; there is no need for amplifiers or other auxiliary equipment and the system is very rugged and reliable. The interconnections necessary for the solution of an equation are carried out by means of short patch cords which can be plugged at both ends into sockets on the panel of the control unit. These sockets are connected to the terminals of the various M-type transmitters and motors in the instrument. The panel contains also a set of bus-bars with multiple outlet sockets, to allow a number of motors to be driven by a single transmitter.

The connections to the plugs are such that a reversal of direction of a motor is achieved simply by reversing one of the plugs in its socket.

Demonstration.

An equation whose solution shows the occurrence of forced oscillations in a non-linear system has been chosen for demonstrating the operation of the instrument. The equation is:

$$\frac{d^2x}{dt^2} + \frac{8}{9}\frac{dx}{dt} + x\left(1 + \frac{x}{20}\right) = 14 \cos t$$

For certain initial values of x and $\frac{dx}{dt}$, the solution shows the presence of sub-harmonics in x. We are indebted to Professor Hartree for providing the initial values * of x and $\frac{dx}{dt}$ which lead to second sub-harmonics in the solution. With these initial values, the solution expressed as a graph of x against sin t, is of the form shown in fig. 2.

In attempting a solution on the differential analyser, it is convenient to express the equation in the form:

$$\frac{dx}{dt} = 14 \sin t - \frac{8}{9} x - \int x \, dt - \frac{1}{20} \int x^2 dt$$

The first step is to draw a schematic diagram to show how the components of the instrument should be interconnected. For simplicity in explanation, we shall omit scale factors and also the constant factors ($\frac{8}{9}$, 14 and $\frac{1}{20}$) appearing in the equation. Fig. 3 shows the instrument set up for the equation:

$$\frac{dx}{dt} = \sin t - x - \int x \, dt - \int x^2 dt$$

In the actual solution, suitable gears were included to take account of the numerical factors; the diagram serves only to illustrate the principle of the solution. The convention used follows that adopted by Bush (4), but it should be remembered that as the interconnections are electrical, the horizontal lines which represent longitudinal

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* $t = 0$, $x = 6.02$, $\frac{dx}{dt} = 12.12$

shafts in the conventional diagram must be interpreted as electrical bus-bars in this case. Connections in the positive sense are represented by solid dots and in the negative sense by circles.

We have explained very briefly how the simple functions, such as addition and integration, are performed in the machine, but we have not yet mentioned the sign of equality. There is no individual component to represent it; it is implied by the interconnection of units, which is such that the various components can rotate only in a manner appropriate to the equation being solved. Further explanation of this point will be made in reference to the problem under discussion.

Let us assume that the rotation of a particular shaft (or electrical bus-bar) represents the variable $\frac{dx}{dt}$. Another shaft, that of the independent drive, represents the independent variable t, and may be made to rotate at any convenient speed.

Since the two variables t and $\frac{dx}{dt}$ are represented by rotations, an integrator may be used to integrate one with respect to the other, and thus to cause a rotation proportional to $\int \frac{dx}{dt}\, dt$, i.e. x. The other functions of x on the right-hand side of the equations can be evaluated by means of integrators and gear boxes. The function, sin t, may be plotted on an input table in terms of t and so fed into the analyser, but it is more convenient to generate it as the solution proceeds, by solving the auxiliary equation:

$$\frac{d^2y}{dt^2} + y = 0$$

which, with suitable initial conditions, yields the solution $y = \sin t$.

The sum of the four terms on the right-hand side is obtained from adding units, and their output must coincide with $\frac{dx}{dt}$ if the equation is to be satisfied. Thus the sign of equality is represented by connecting the output shaft representing the right-hand side of the equation to the input shaft which was assumed to represent the left-hand side, $\frac{dx}{dt}$. The various shafts in the instrument can then rotate only in such a way that the quantities they represent are consistent with the equation being solved. The solution is obtained by connecting the shafts representing the appropriate variables (x and $\sin t$ in this case) to an output table.

Fig. 3 shows schematically how this is achieved. Bus-bar 1, independently driven, represents the independent variable t. Bus-bar 9 will be assumed to represent $\frac{dx}{dt}$. Integrator V receives its input from those two bus-bars, and its output is fed to bus-bar 4, representing x. Integrator III provides the value of $\int x\,dt$ for bus-bar 5, and using this integral as the argument and x as the integrand, Integrator IV evaluates $\int x\,d\left[\int x\,dt\right] = \int x^2 dt$, which is fed to bus-bar 6.

Integrators I and II are used for solving the auxiliary equation $\frac{d^2y}{dt^2} + y = 0$, $y = \sin t$ appearing on bus-bar 3.

The quantities represented on bus-bars 3, 4, 5 and 6, are the four terms on the right-hand side of the equation. These are added in pairs, with due regard to sign, by means of three adding units, and the output from this operation represents the right-hand side of the equation, which must be equal to $\frac{dx}{dt}$. The sign of equality is thus represented by feeding the output of the addition (i.e. the right-hand side of the equation) directly to bus-bar 9 (i.e. $\frac{dx}{dt}$,

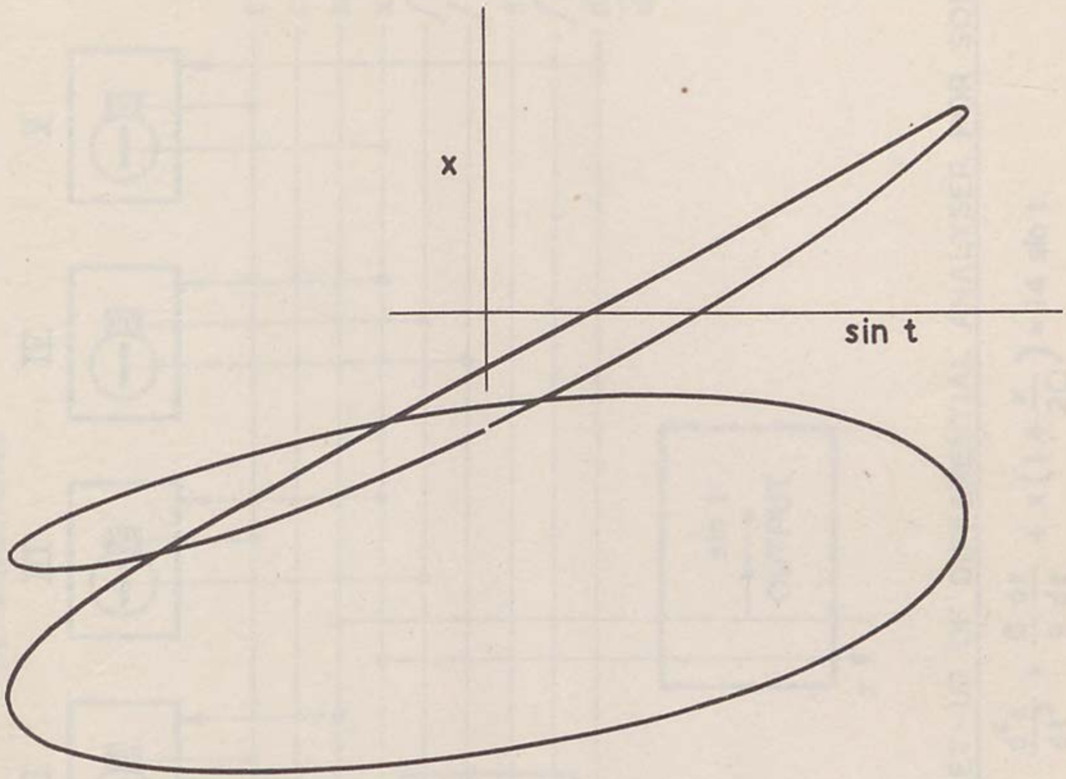


FIG. 2. SOLUTION OF $\frac{d^2x}{dt^2} + \frac{8}{9}\frac{dx}{dt} + x\left(1 + \frac{x}{20}\right) = 14 \sin t.$

FOR $x = -6 \cdot 02$; $\frac{dx}{dt} = 12 \cdot 12$ AT $t = 0$.

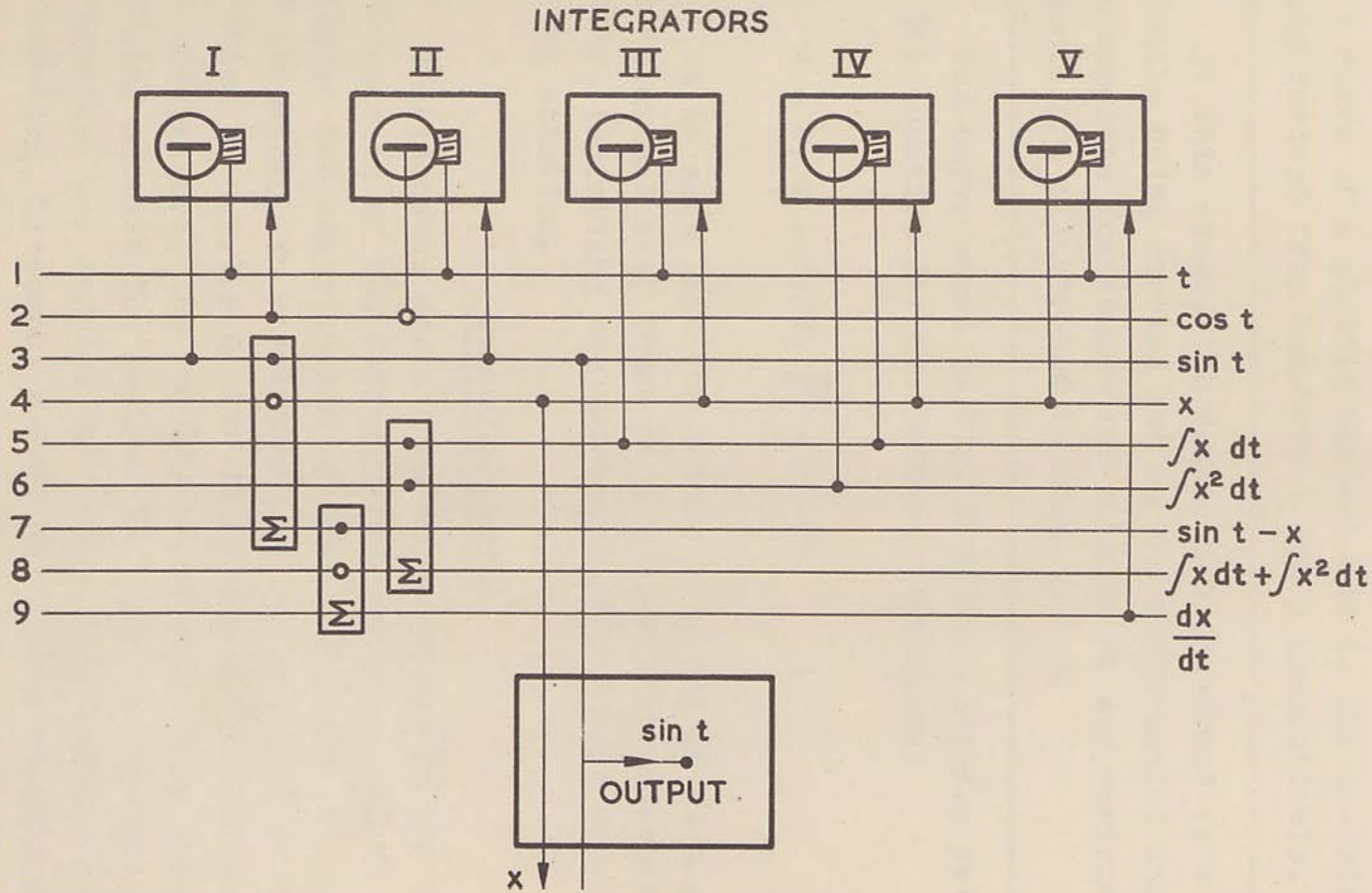


FIG. 3. SCHEMATIC SET-UP OF DIFFERENTIAL ANALYSER FOR SOLVING

$$\frac{d^2x}{dt^2} + \frac{8}{9}\frac{dx}{dt} + x\left(1 + \frac{x}{20}\right) = 14 \sin t.$$

c
r
a
D
t
c
i
C.
th
N.
St

the left-hand side of the equation).

The output table presents the solution in the form of a curve of x plotted against sin t, the co-ordinates being derived from bus-bars 4 and 3 respectively.

At this stage, the conference adjourned to see the solution being carried out on the differential analyser. The complete instrument and details of its various components are illustrated in figs. 4 - 9.

The curve shown in fig. 2 is the analyser solution of the equation for the initial conditions:

$$t = 0, \ x = 6.02, \ \frac{dx}{dt} = 12.12$$

It is seen that the values of x and $\frac{dx}{dt}$ at $t = 4\pi$ are identical with those at $t = 0$, within reasonable limits, indicating the existence of a second sub-harmonic in the solution.

Acknowledgments

Considerable assistance was received in the design and construction of the differential analyser, and we wish to acknowledge the contribution of many colleagues in C.S.I.R.O. and elsewhere. In particular, we are grateful to the Metrology Division of C.S.I.R.O. for undertaking the tedious tests on the integrators, the results of which gave us confidence in carrying on with the design. The detailed design of the integrators was carried out by the Commonwealth Aircraft Corporation, Lidcombe, N.S.W. and they were manufactured by that body and by the Government Small Arms Factory, Lithgow, N.S.W. The drawing office and workshop of the National Standards Laboratory took a major part in the project.

the left-hand side of the equation).

The output table presents the solution in the form of a curve of x plotted against sin t, the co-ordinates being derived from the bars a and β respectively.

At this stage, the conference adjourned to see the solution being carried out on the differential analyser. The complete instrument and details of its various components are illustrated in Figs. 4 - 8.

The curve shown in Fig. 2 is the analyser solution of the equation for the initial conditions:

$$t = 0, \ x = 5.02, \ \frac{dx}{dt} = 12.42$$

It is seen that the values of x and $\frac{dx}{dt}$ at $t = 4\pi$ are identical with those at $t = 0$, within reasonable limits, indicating the existence of a second sub-harmonic in the solution.

Acknowledgements

Considerable assistance was received in the design and construction of the differential analyser, and we wish to acknowledge the appreciation of many colleagues in C.S.I.R.O. and elsewhere. In particular, we are grateful to the Metrology Division of C.S.I.R.O. for undertaking the tedious tests of the integrators, the results of which gave us confidence in carrying on with the design. The detailed design of the integrators was carried out by the Commonwealth Aircraft Corporation, Lidcombe, N.S.W., and they were manufactured by that body and by the Government Small Arms Factory, Lithgow, N.S.W. The drawing office and workshop of the National Standards Laboratory took a major part in the project.

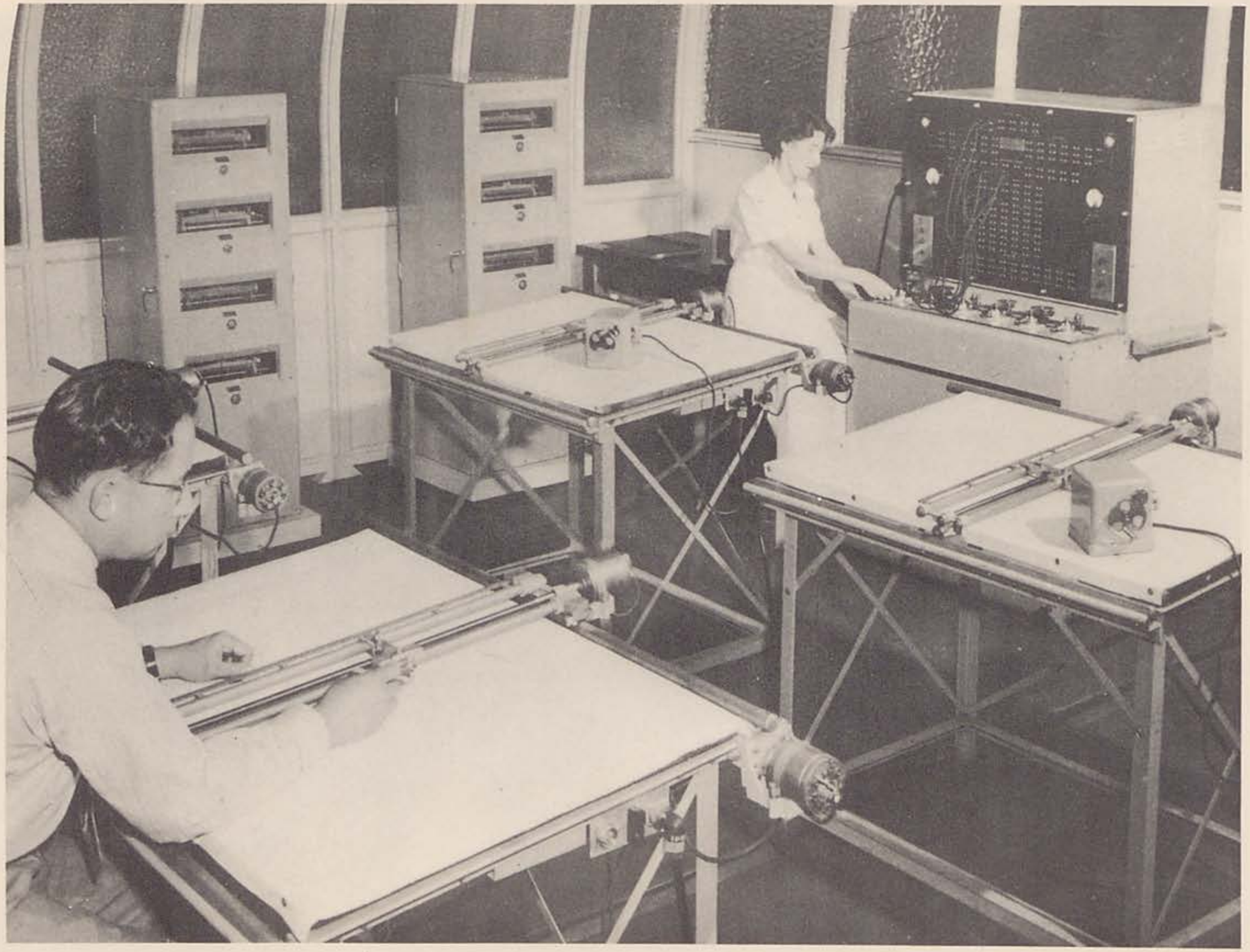


Fig.4 - General view of C.S.I.R.O. differential analyser

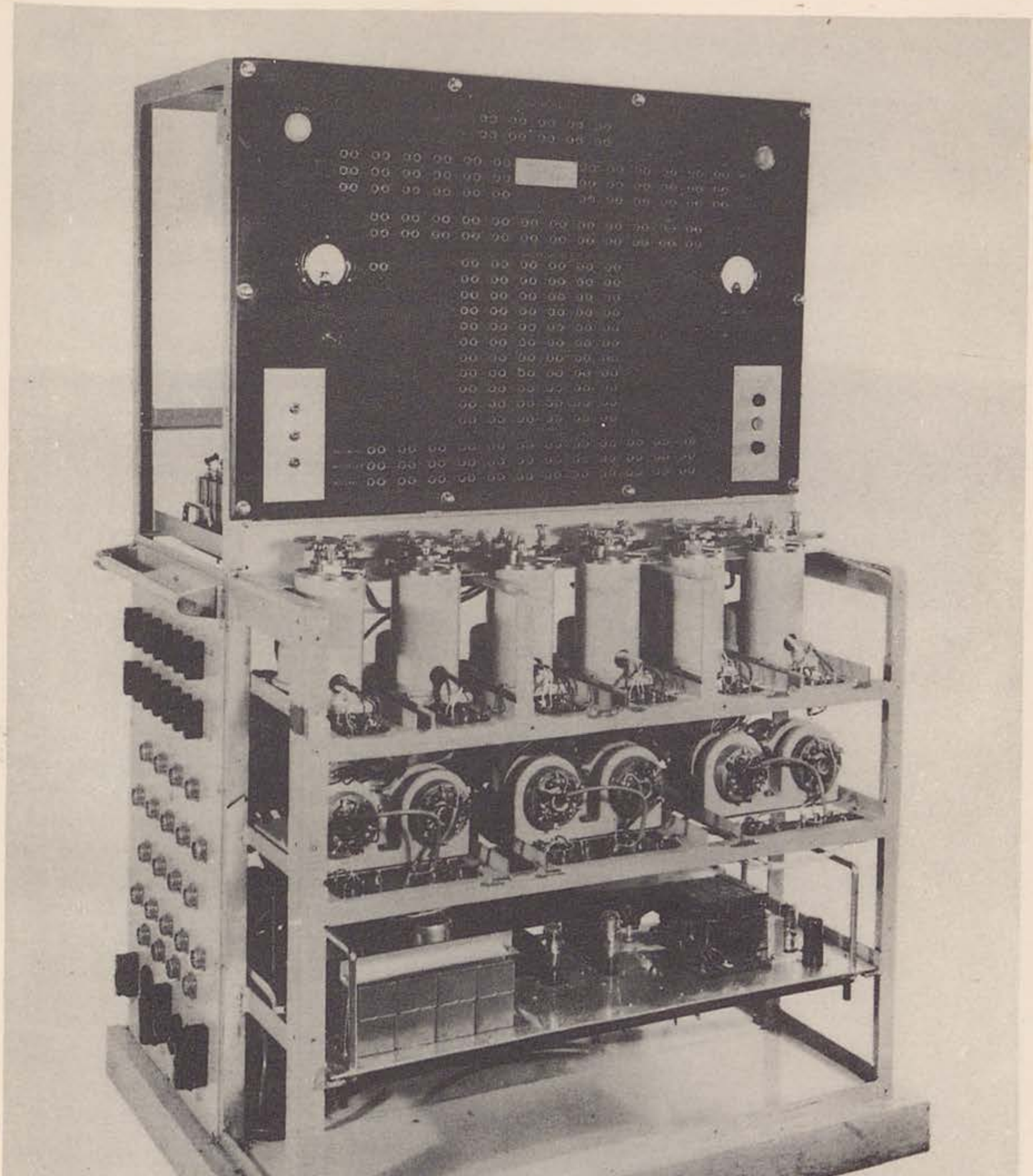


Fig.5 - View of main control unit showing control board, adding and ratio units. (Covers removed)

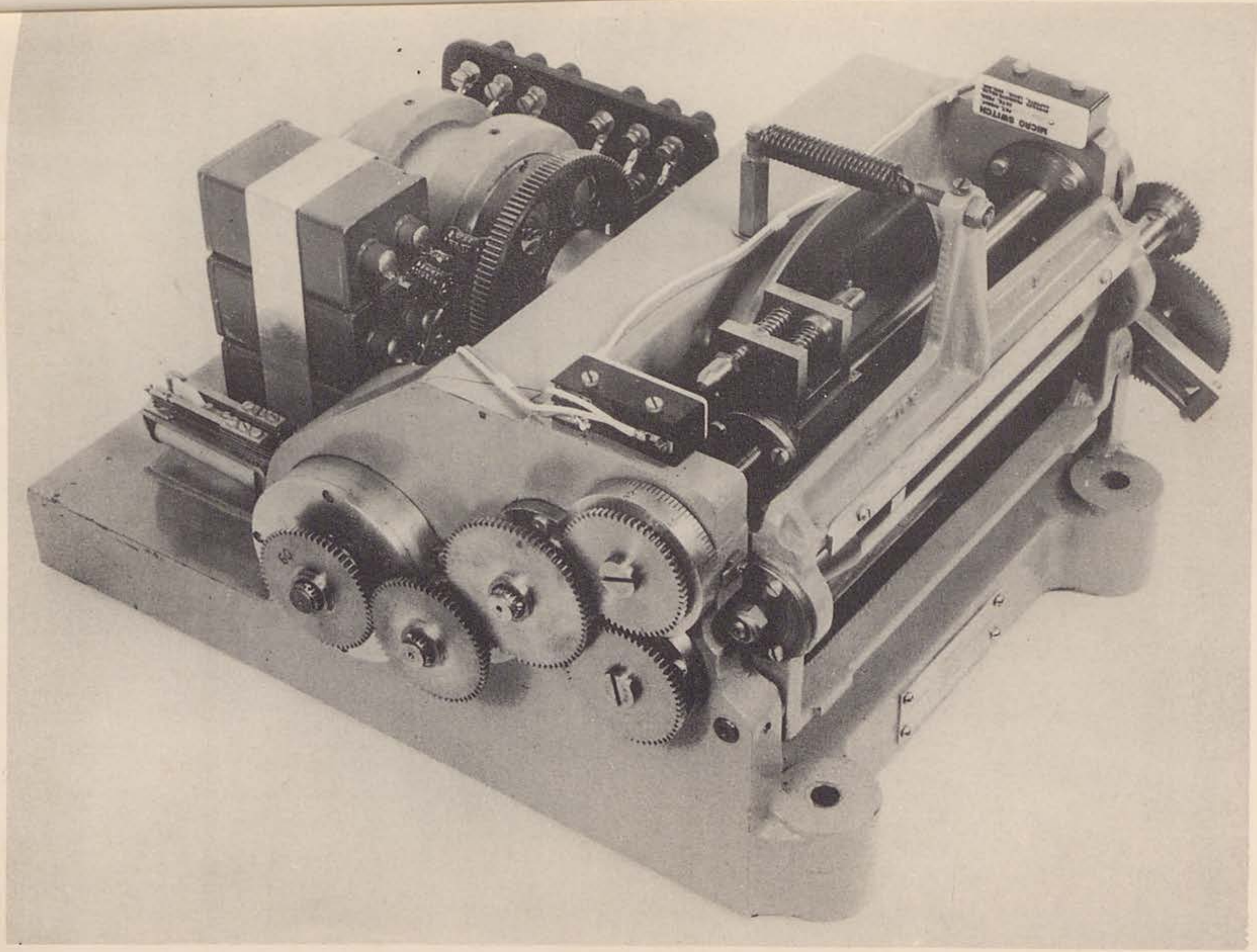


Fig.6 - Integrator showing integrand gear train

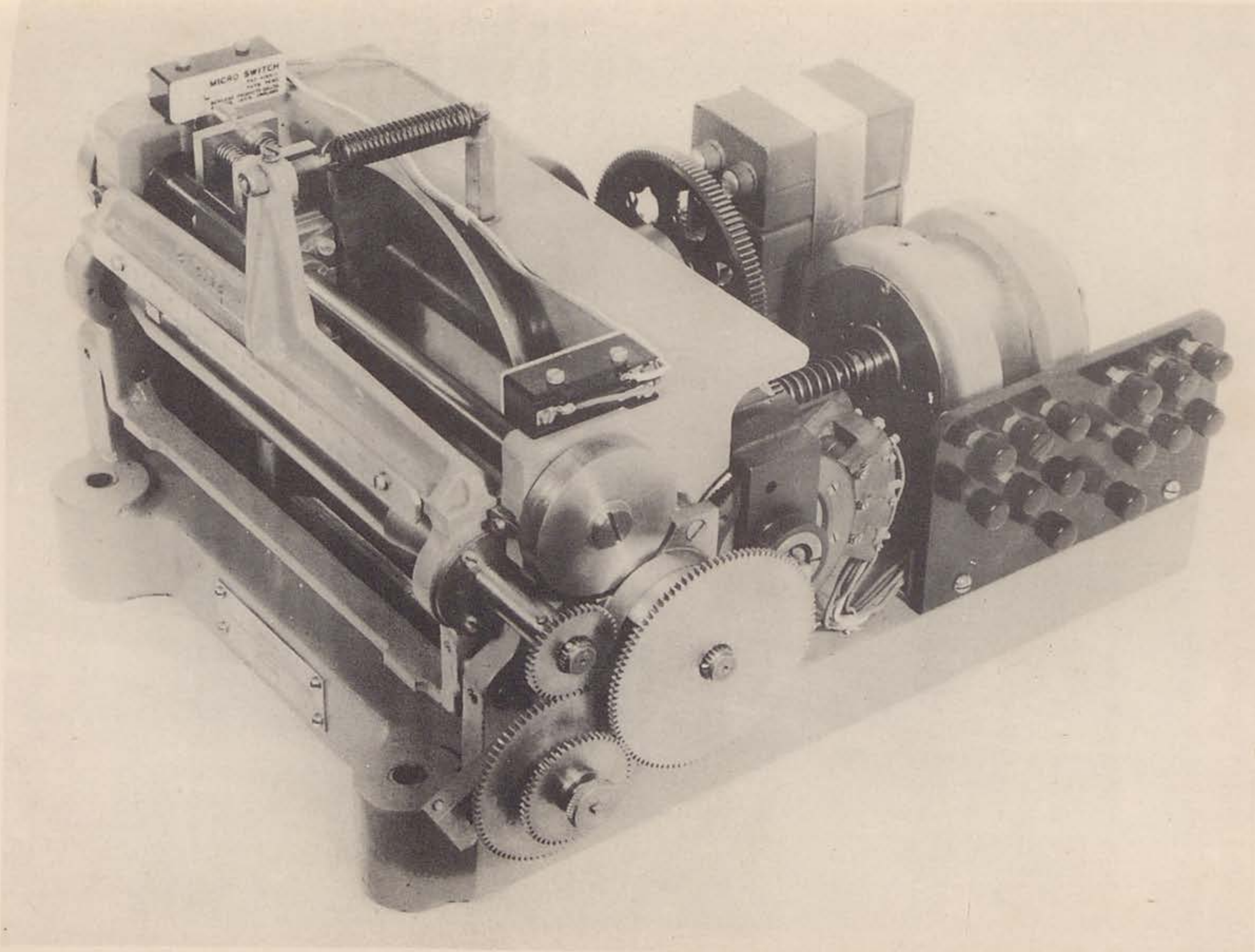


Fig.7 - Integrator showing output gear train

Fig. 8 - Ratio unit

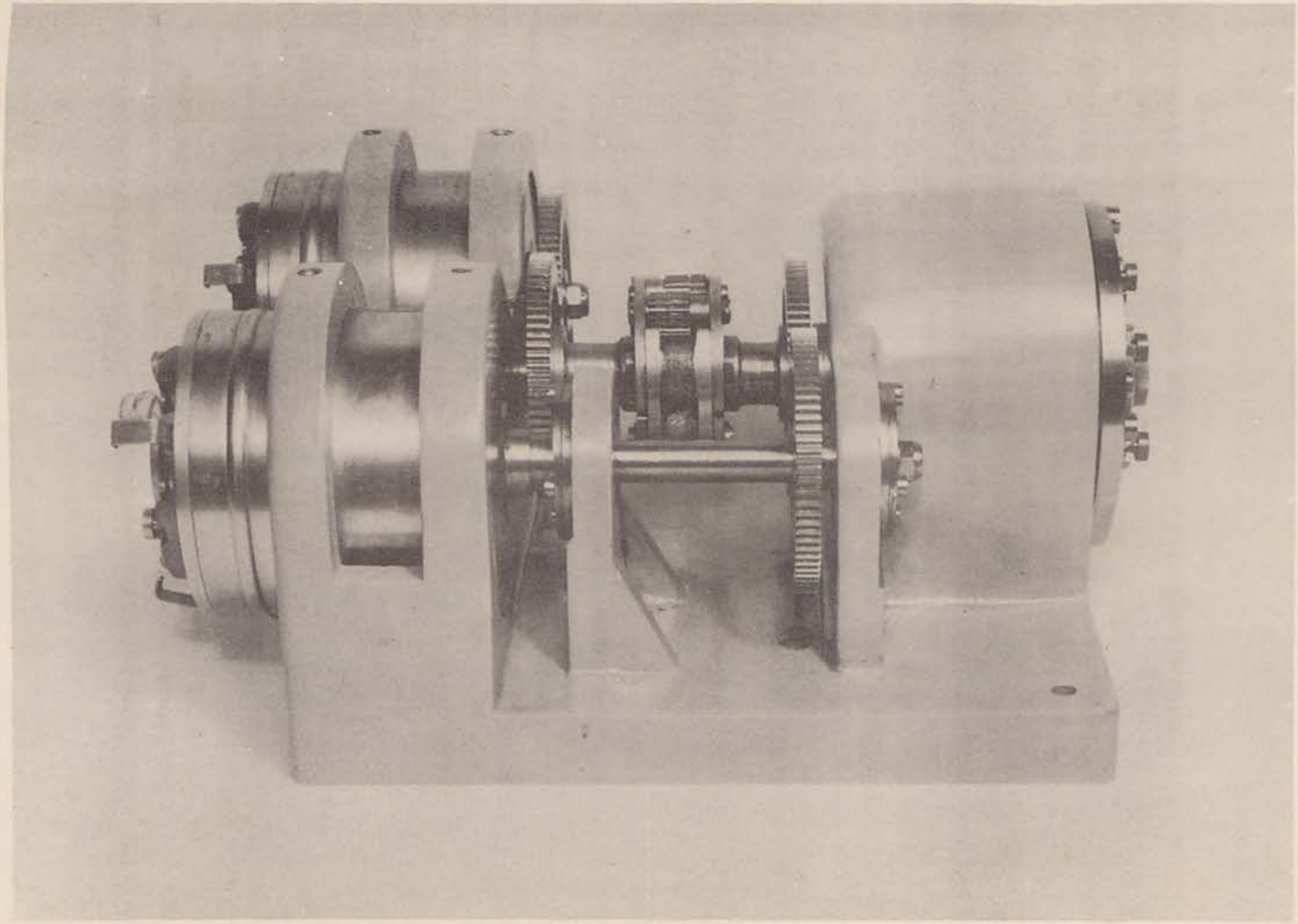


Fig.9 - Adding unit

III Automatic Digital Calculating Machines

by D. R. Hartree

1. Some notes on terminology

The subject of automatic digital calculating machines is a new one, and any new subject of significance involves new ideas for which we require words. These may either be new words coined for the purpose, or words already in current use, given a specialised meaning. In this subject the latter alternative is generally adopted, but although there is some general agreement, the terminology has not become standardised, and rather naturally each group prefers the terminology which it has built up and found convenient.

The part of the machine which holds numbers and operating instructions is usually called the "store" or "memory". To avoid misunderstanding I think it is desirable to avoid the use of words with biological implications, and for this reason I much prefer the neutral - and shorter - word "store", which incidentally was used in this sense by Babbage.

The specification to the machine of an operation to be carried out, such as the transfer of a number from the arithmetical unit to the store, I shall call an "operating instruction" or an "instruction". Other terms for this are "order" and "command"; the former is the general usage at Cambridge and the latter at the C.S.I.R.O. group at Sydney.

The schedule of instructions for carrying out a calculation is usually known as the "program" for the calculation, and the process of drawing up this program is known as "programming". A distinction has sometimes been made (by myself among others) between "programming" and "coding", the former being the planning of the sequence of operations required to carry out a calculation, and the latter being the process of expressing these operations in terms of the standard kinds of instructions for a particular machine. But I do not now think that this distinction is a useful one in practice, at any rate for machines using instructions of a simple form. In simple applications of such machines, one does not in fact carry out separately the two steps expressed by the terms "programming" and "coding" in this sense; one programs the calculations directly in terms of the operating instructions in the form in which they are furnished to the machine. And it is usually possible to break down a more involved calculation into a number of component parts each of which is simple enough to be programmed directly in this way.

2. Some general considerations.

Automatic digital machines have two aspects, one concerned with their design and construction, and the other with their use. These are not wholly independent, but it ought to be possible to use one of these machines without knowing in detail how it does what it does, just as one is accustomed to use a dial telephone without knowing the details of the circuitry of an automatic telephone exchange by which dialling a number results in the required connection being made. To the potential user, with numerical calculations to be done for which he would like the assistance the machine can provide, the second aspect is at least as important as the first. The provision of a machine is only the first step; equally important is the provision of adequate facilities for using it. With this aspect I shall be concerned later (§ 6 and V.)

Let me first recall the general organisation of an automatic digital machine (I, §3). It must comprise a store, for numbers and for operating instructions, an arithmetical unit, facilities for transfer of numbers between the store and the arithmetical unit, and a control unit which organises these transfers and determines the operations carried out by the arithmetical unit on numbers transferred to it. The machine must also have an input unit for receiving from the outside world operating instructions and numerical data, and an output unit for furnishing to the outside world the results of the calculations it has carried out.

As already mentioned (I, §4), in most of the more recent machines numbers and operating instructions are represented in the same form within the machine, and the same store is used for both. This store can be regarded as consisting of a number of storage locations in which numbers or instructions can be placed; these storage locations must be identifiable in some way, and it is convenient to distinguish them by assigning to each a number, known as its address or, more usually, the address of its content. The content of a storage location is usually called a "word", when we want to refer to it without specifying whether it is a number or an instruction. The difference between "words" representing numbers and "words" representing instructions in such a machine lies in the way they are used, "words" representing numbers being transferred from the store to the arithmetical unit for arithmetical operations to be carried out on them, and back to the store, whereas "words" representing instructions are transferred from the store to the control unit, where they determine the transfers carried out between the store and the arithmetical unit, and the operations carried out in the arithmetical unit. Numbers consisting of more digits than can be included in a single word are usually called "double-length" or "multi-length" numbers.

As already mentioned (I, §4), use of the same form, within the machine, for numbers and for instructions has two consequences. First, the same store can be used both for numbers and instructions so that the whole storage capacity can be partitioned between numbers and instructions in the way most appropriate for each calculation. Secondly, and much more important, since there is no longer any distinction within the machine between numbers and instructions, the facilities of the arithmetical unit can be used to modify the operating instructions themselves in the course of a calculation, this being done quite automatically, as the result of other instructions. The importance of this can probably not be fully realised without some experience in programming, but must be accepted as a conclusion from experience; it introduces what might be called an extra degree of freedom into the process of programming, and makes this process one of great flexibility and versatility. On account of the importance of this feature I shall restrict myself to machines which do provide it.

3. Representation of numbers in the machine.

In our usual way of writing numbers in digital form, the number 3807, for example, stands for

$$3\;8\;0\;7$$

$$3 \times 10^3 + 8 \times 10^2 + 0 \times 10 + 7$$

the successive digits being coefficients of successive powers of ten; this is called the "decimal" or "scale-of-ten" form of a number. But there is no particular merit, other than that of familiarity, in the use of ten as a base, and a convenient alternative base is the number two; then the only possible values for each digit are 0 and 1, and the number 1101, for example, stands for

$$\begin{array}{c} 1\ 1\ 0\ 1 \\ 1 \times 2^3 + 1 \times 2^2 + 0 \times 2 + 1 \end{array}$$

that is, an eight, a four, no twos and a one; the number which in our decimal notation we write 13. This is called the "binary" or "scale-of-two" form of a number. Use of the binary form of numbers within the machine does not mean that the user need have any working experience of arithmetic with numbers expressed in this form. In using the machine almost the only operations for which it is necessary to remember that numbers are expressed in the machine in binary form are those concerned with left and right shifts; a left shift of one place corresponds to multiplication by two not by ten, and a right shift of one place corresponds to division by two. Since multiplication and division by two are operations often required in the course of a calculation, whereas multiplication and division by ten are required relatively seldom, this is a convenience rather than a disadvantage of the use of the binary form. Although numbers are represented in binary form within the machine, they can be input in decimal form, and the results recorded in decimal form, the decimal-binary and binary-decimal conversions being carried out by the machine as part of the input and output processes.

Even though a machine is designed normally to work with numbers in binary form, it is quite possible to program it to work with "words" representing numbers expressed in scale of ten - or scale of seven or of twelve or of a thousand if required - suitably coded; this may be convenient in some contexts, in particular in working with multi-length numbers. I used to think that there was an advantage in having numbers represented in some coded decimal form within the machine, on the ground that they could then be easily displayed in decimal form, which would be useful both in testing the machine and diagnosing any faults in its operation, and in testing and checking a program by making the machine work in steps of one instruction for each operation of a control key and watching the display of results on some monitor system, a procedure sometimes called "peeping". But for testing the machine this use of a display in scale of ten has been found to be quite unnecessary, and for testing and checking programs, "peeping" is a procedure which is so extravagantly wasteful of machine time that, if the machine is at all fully occupied, it should be most strongly discouraged; its place should be taken, as far as possible, by a system of testing and checking programs which uses the machine running at its normal operating speed, recording sufficient intermediate results for the guidance of the programmer. An account of some such methods for testing programs has recently been given by S. Gill (11) (see also ref. 57).

Negative numbers can be represented by a sign indication and either the modulus or the complement of the number. Most machines use the complementary form, with the sign indication in the most significant digital position, a zero indicating a positive sign and a 1 indicating a negative sign.

4. Representation of instructions.

There are two main forms for the operating instructions, which I shall illustrate by an example.

I shall write C(n) for "the content of storage location n", so that n stands for an address in the store and C(n) for the word, whether number or instruction, which happens to be located there at the relevant stage of the calculation. Suppose now that we want the machine, as part of a larger calculation, to add the contents of storage locations 82 and 84 and to send the result to storage location 122. This could be specified, in some coded form, by a single instruction which we might write

C(82) + C(84) to 122

Such an instruction refers to three addresses in the store, and this form for instructions is therefore called a "3-address form".

Alternatively, we could obtain the same result by three separate instructions each referring to a single address in the store. Suppose the arithmetical unit contains a component, usually called an "accumulator", which accumulates the sum of numbers added into it until an instruction is given to clear it; I shall write C(Acc) for the content of the accumulator. Then the addition sum mentioned could be carried out by the three instructions

C(82) to Acc
C(84) to Acc
C(Acc) to 122,

taken successively. Since each of these only refers to one address, this form for instructions is known as a "one-address" form.

But it is not enough to specify to the machine an operation to be carried out; it is necessary to enable the machine, once this operation has been completed, to determine the next instruction. This can be done in two ways. One is to include in each instruction a specification of the address from which the next instruction is to be taken. If this is combined with a three-address form of specification of the current operation, we get altogether a four-address form of instruction, three addresses being concerned with the current operation and one with the selection of the next instruction.

Alternatively, the instructions can be stored normally at addresses numbered in the sequence in which they are to be carried out. Then a counter whose content is increased by unity at the completion of each instruction can be used to determine the address from which the next instruction is to be taken, without this having to be mentioned explicitly in each instruction. This is called "serial storage" or "sequential storage" of instructions, and the counter whose content records the address from which the current instruction was taken is called the "sequence control register". An explicit specification of the address from which the next instruction is to be taken is required only when one wants the machine to depart from the sequence in which instructions are held in the store, and a special kind of instruction is necessary for this.

To the user of a machine, the important things about it are not the kind of storage system used or the form in which numbers are represented in the machine, but the general form of instructions, the standard kinds of instruction, and the way of determining the next instruction, for which the control system is designed. Any calculation must be programmed in conformity with these standard instructions and this way of determining the next instruction, and these are the aspects of a machine which matter most to the programmer.

5. Organisation of a machine using a one-address form of instructions with sequential storage.

Fig. 1 shows in schematic form one kind of organisation for a machine using a one-address form of instructions with sequential storage; this is based on the EDSAC at the Mathematical Laboratory of the University of Cambridge (56). The different units of the machine are shown connected to two bus lines * through gates, marked by crosses, which are normally closed but can be opened by the operation of the control system. These gates can be divided into two groups, "source" (S) gates through which "words" can be transferred from the different units, and "destination" (D) gates through which "words" can be transferred to them. Most operations of the machine involve the opening of one S gate and one D gate, by which the transfer of a "word" from one part of the machine to another is effected.

Concerned with the control of the sequence of operations are shown two registers, the "current instruction register" ("c.i. register") for holding the current instruction itself, and the "sequence control register" ("s.c. register") for holding the address from which the current instruction was taken. The operation of this machine can be thought of as

* Forming a "digit trunk" in the terminology of the C.S.I.R.O. group at Sydney.

having two stages which take place alternately. Normally, at the end of the operation specified by one instruction, gate A is opened and the content of the sequence control register is increased by unity, giving the address from which the next instruction is to be taken. In the first stage of operation, this address is sent to the control system, which then opens the appropriate S gate in the store and the D gate "D Inst" to permit the transfer of the instruction located at that address in the store to the current instruction register. In the second stage, this instruction itself is sent to the control system, and opens the appropriate gates for the instruction to be carried out; and on completion of it, gate A is opened and the content of the sequence-control register is again increased by unity. Thus the machine alternately selects an instruction and carries it out. A special instruction is necessary to make the machine depart from the serial order in which instructions are stored; this instruction, when transferred to the control unit, results in gate J being opened instead of gate A, and then the address specified in the current instruction is transferred to the sequence control register and determines the address from which the next instruction is taken.

6. Programming

The individual operations which such a machine carries out are very simple ones, such as addition, subtraction and multiplication of two numbers, transfer of a number from the accumulator of the arithmetical unit to the store, and selection of the next instruction; and any calculation must be broken down to a sequence of such operations before the machine can be applied to it. The schedule of

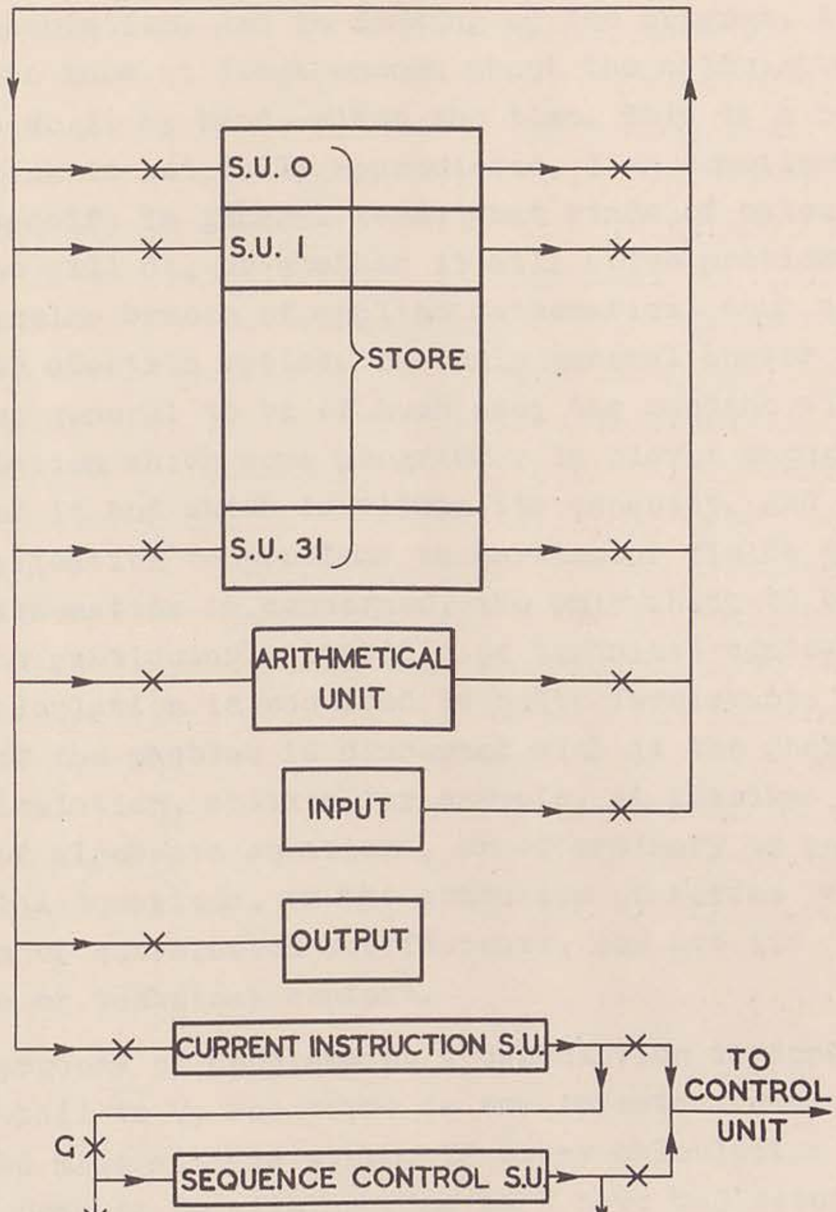


FIG. 1. SIMPLIFIED FORM OF GENERAL ORGANIZATION OF THE E.D.S.A.C. CROSSES INDICATE GATES OPERATED BY CONTROL UNIT.

instructions for such operations is known as the <u>program</u> for the calculation, and in drawing up the program, it is necessary to know at least enough about the calculation to be able to do it by hand, given the time. This is a point which I think is not fully appreciated. I am sometimes asked to specify in general terms what kinds of calculation the machine will do, or whether it will solve problems in some particular branch of applied mathematics, such as fluid dynamics or electron optics. The only general answer I can give is too general to be of much use; the machine will do any calculation which some programmer is clever enough to program for it and which is within its capacity. And as far as its application to problems in particular fields of applied mathematics is concerned, the only thing to be said is that the particular scientific or technical context in which a calculation is required is quite irrelevant; what the user of the machine is concerned with is the character of the calculation, whether, for example, it involves the solution of algebraic equations, or of ordinary or partial differential equations, or the summation of series or the evaluation of correlation coefficients, and not its scientific or technical context.

The process of programming a calculation is considered in some detail in V, but there is one important general point to be made at this stage. If every calculation had to be broken down to details as fine as I have indicated, programming would be a long and tedious process, and also one which would require extensive checking. But most extensive calculations can be built up to a large extent from groups of operations for standard processes such as evaluation of square roots and trigonometrical functions and their inverses, interpolation, and integration. The group of instructions for such a process is known as a <u>sub-routine</u>.

One kind of organisation for simplifying programming is based on the provision of a library of such sub-routines for standard processes, and facilities for using it. The library sub-routines can be worked out and thoroughly checked once for all, and the program for a whole calculation may then consist of a set of sub-routines, perhaps some from the library and some made up specially for the particular calculation, and a <u>master routine</u> which is concerned largely with organising the sequence in which the sub-routines are taken. Use of library sub-routines both saves much time, labour and thought in programming a calculation, and it also saves many of the mistakes which might be made in programming the calculation in full, and also the programmer's and machine's time which might be spent in diagnosing and correcting these mistakes. The development of such a library is not a small matter, but it is something which is likely to be required before one of these machines becomes practically effective as a general-purpose calculating machine, and this must be recognised by anyone who is contemplating constructing or obtaining such a machine.

7. Non-numerical applications.

I have spoken so far as if the only things these machines can do were numerical calculations; but they can do other things as well - or to put it another way, there are some problems which we do not normally regard as numerical but which can be put into numerical form and so made accessible to the machine.

For example, there is a simple puzzle of this form. There is a row of nine holes, of which the right-hand four are initially occupied by black marbles and the left-hand four by white marbles. Allowed moves are as follows: for a black marble a move one place to the left into a vacant space, or a jump to the left over a single white marble into a vacant space, and for the white marbles similar moves to the right. The object is to interchange the black and white marbles, making only the allowed kinds of moves. It is possible to program the machine to work out how to do this, and to print out the sequence of moves.

This is a rather simple example, but will serve as an illustration of one kind of non-numerical problem to which the machine can be applied.

IV Digital Calculating Machines Used by C.S.I.R.O.

by T. Pearcey and M. Beard

Some three years ago the Division of Radiophysics of C.S.I.R.O. commenced a programme of development and investigation into the design and use of automatic computers and computing methods to aid in computations for scientific research. This programme has two aspects.

First, commercial calculating machines of the punched card type were installed some two years ago and have been in operation since that time following certain adaptations which were made to them by ourselves with the approval of the British Tabulating Machine Company. These adaptations were intended to improve the performance of the machines for purposes of scientific computing. These machines have been steadily at work and during the past year have encountered fresh work at a greater rate than it can be performed. Amongst the computations made are time series and spectral density estimations, crystalographic Fourier syntheses, the tabulation of astronomical and mathematical tables of various kinds, reduction of observations and the solution of integral and differential equations as well as the solution of sets of linear algebraic equations.

The second and main aspect of the Division's programme has been in the development of an electronic automatic digital computer and I will concentrate upon a description of that device.

This computer, known so far as the "Radiophysics Mk. I Automatic Computer", is of the all-electronic digital type, and operates in the binary scale. It is of the serial type, that is, data are transferred from point to point of the machine in a serial manner, or a digit by digit manner, commencing the transfer with the lowest significant digit and ending with the most significant digit.

All transfers of digits take the form of a train of electrical impulses, carried over conducting cables, which coincide with the pulses of a regular train of pulses generated by a control timing device or "clock".

In the binary scale numbers are represented by a series of interspersed zeros and units. In the computer a unit is represented by a pulse and a zero by the absence of a pulse.

2. Numbers and commands

Both commands and subject matter, or numbers, are stored in the computer in the same manner. The length, that is the number of digits, comprising a command is the same as that for a standard number, namely 20 digits. A command looks like a number in the machine, it is however adopted for use by the control system of the computer in a special manner and according to a predetermined convention which is implied by the design of the machine.

A number is a single group of 20 binary digits. Each digit is given a weighting factor equal to an integral power of two. Digits of a number are read on a wire just as a number, normally written, would be read from right to left, not from left to right. Such a scheme simplifies the arithmetical operations.

Negative numbers are stored in a "complementary" form, that is, stored with all their units replaced by zeros, and their zeros replaced by units, and a unit is added to the lowest significant place. This is illustrated in fig. 1.

The appearance of the lowest significant digit

INTEGERS

Dec.	Bin.
0	0
1	1
2	10
3	11
4	100
5	101

POSITIVE NUMBERS

$$1101\cdot0100 = 8+4+1+\frac{1}{4} = 13\cdot25$$

$$0011\cdot0101 = 2+1+\frac{1}{4}+\frac{1}{16} = 3\cdot3125$$

$$= 13\cdot25 \div 4$$

FRACTIONS

Bin.	Dec.
0·1	1/2
0·01	1/4
0·11	3/4
0·001	1/8
0·101	5/8

Binary Point

NEGATIVE NUMBERS

Replace 1's by 0's

0's by 1's

Add Fugitive Digit

ADDITION

1 + 1 = Carry

1 + 0 = 1

$$-3\cdot3125 = -0011\cdot0101$$

$$= (1)1100\cdot1010$$

Add

$$= 1100\cdot1011 \text{ (Mod. 16)}$$

= Complement of 3·3125

SUBTRACTION

Add

Complement

FIG.1. BINARY ARITHMETIC (1 + 1 = 10).

corresponds to a clock pulse denoted by p_1 and succeeding digits correspond to the symbols p_2, p_3, etc. up to p_{20}. The p_{20} digit of a number corresponds to the sign digit, zero if positive, unity if negative.

A command however is considered for the purpose of the computer to consist of three adjacent components. Two of these components occupy the digit groups p_1 to p_5 and p_6 to p_{10}. The third occupies the group p_{11} to p_{20}.

The first two groups serve the purpose of defining the storage registers which are to transmit and receive data and to specify any arithmetical functions to be performed. The last group is devoted to indicating which position in the store is to be called upon for action if needed. Whilst the first two "addresses" specify a "source" and a "destination", the third is purely a subsidiary address.

3. Sequential Arrangement

For the purpose of organising transfers of numbers and commands to and fro within the computer the regular train of clock pulses is broken up into groups. A single clock period is three μ secs. long and contains a pulse of $\frac{1}{2}$ μ sec. length. Twenty such pulses cover a period of 60 μsecs. known as a minor cycle, equal to the length of a number or command. Sixteen such successive periods constitute a major cycle. A major cycle is the longest period into which digit trains are compiled. These periods are illustrated by fig. 2.

4. Storage

The main or high speed store follows principles which are well known, namely the use of ultra sonic acoustic delay tubes. This storage system consists of a number of independent closed channels in each of which 16 numbers or commands can be stored end to end in a circulating manner. The circulation time is equal to one major cycle, namely one millisecond.

Up to 64 channels are available incorporating 32 acoustic tubes filled with mercury and fed by 10 mcs amplitude modulated pulses applied to resonant X-cut quartz crystals. The acoustic waves are detected at the far end of the tubes by similar crystals. The signal is amplified and the rectified output is used to "gate" a fresh clock pulse back into the transmitting crystal and so on.

Numbers may be "read" out of any of the 16 positions of any one circulation network during a minor cycle interval, each major cycle, depending upon the actual position required. Fig. 3 illustrates the acoustic circulation scheme, whilst fig. 4 shows the actual tubes in position together with amplifiers on the left, gates for reading out and substituting fresh data in the centre, and pulse transmission equipment on the right. The tubes are about five feet long. In all 1024 separate numbers can be stored by this means. An auxiliary store is also incorporated. This consists of a "magnetic drum". This is a cylinder, which revolves at a rate of 6000 r.p.m., coated with a magnetisable surface and capable of storing 1024 axially placed rows of digits around its circumference at a density of 80 words per circumferencial inch.

The access time for any one row of digits, i.e. a word, is equal to 10 milliseconds. The drum is not

PERIODS.

1. *'Clock' Period: 3 μsecs.*
   Pulses: 333 Kc/s of ½ μsec. length.

2. Minor Cycle : 20 Clock Periods of 60 μsecs.

3. Major Cycle : 16 successive Minor Cycles.

NUMBERS & COMMANDS.

NUMBERS
- As 19 Binary Digits & Sign Digit.
- Negative Numbers as Complements.
- Always read Right to Left.
- Presence of Pulse denotes Unit.

COMMANDS
- As 20 Binary Digits.
- 10 Digits detail Transfer Gates.
- 10 Digits purely numerical.

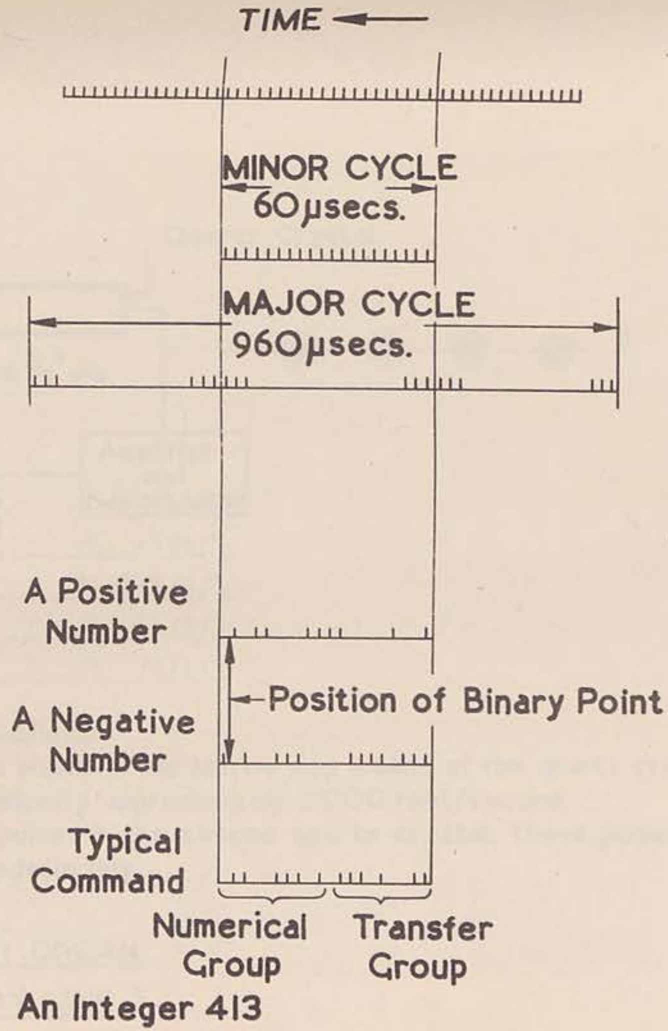
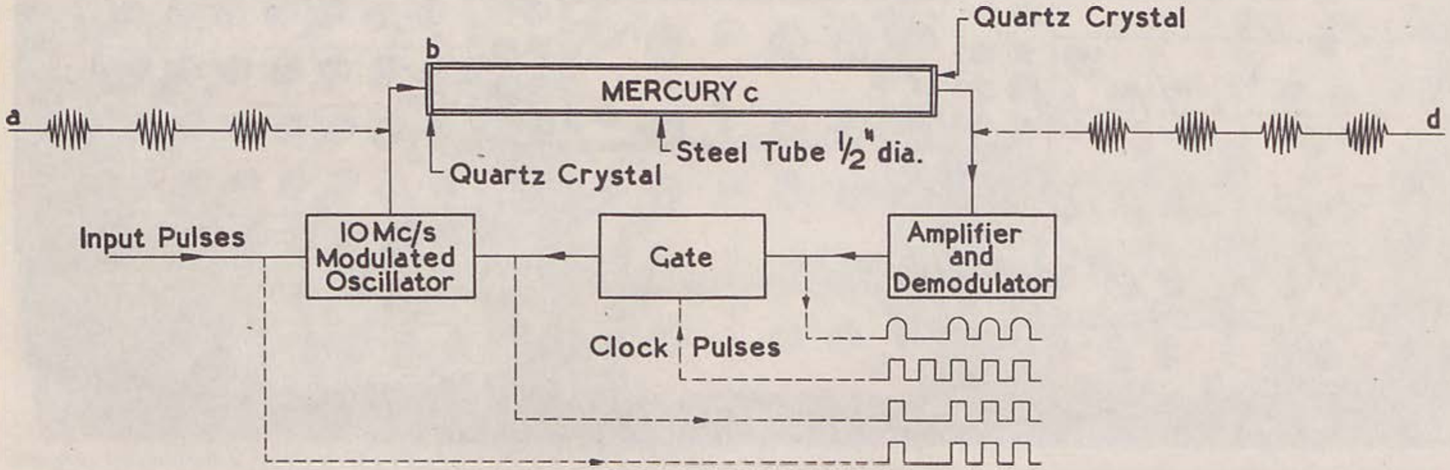


FIG. 2. SEQUENTIAL ARRANGEMENT.

I. Regeneration and Circulation of Pulses in Delay Line.



a) The pulses of an incoming number modulate a 10 Mc/s oscillator.
b) The radio frequency pulses are converted to ultrasonic pulses in the Mercury by means of the quartz crystal.
c) The ultrasonic pulses travel through the Mercury at a velocity approximately 5000 feet/second.
d) The ultrasonic pulses are converted to radio frequency pulses in the second quartz crystal. These pulses are reformed in a gate circuit and may be circulated indefinitely.

FIG. 3. MEMORY ORGAN.

(MERCURY DELAY LINE).

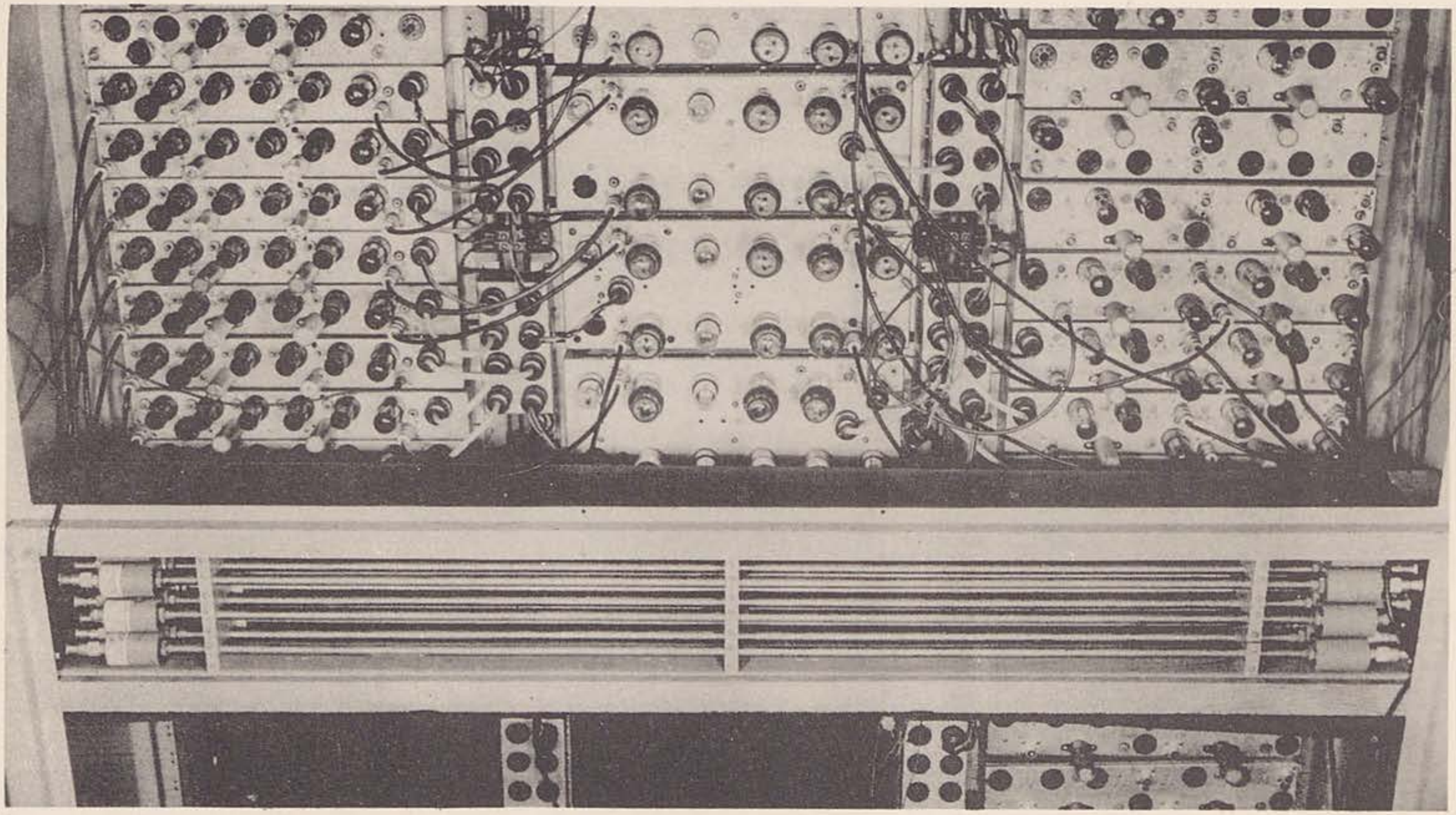


Fig.4 - Delay line storage tubes

synchronised with the speed of the main computer but possesses its own timing device which is coupled to its speed of rotation. Fig. 5 illustrates the principle upon which the drum operates.

The capacity of the store is to be extended to 4096 words, to be grouped into four groups of 1024 in number.

5. General organisation of the computer.

All commands or instructions are held during operation in the main store or acoustic memory and are dealt with serially. That is they are placed into store positions in the serial order in which they will normally be accepted.

The computer consists essentially of a group of registers which are capable of receiving and/or transmitting their contents when ordered to do so. Transmission of data is made via a single channel known as the "digit trunk". All registers are connected to the digit trunk via their "function gates". These "gates" allow digits to pass to or from the registers and the digit trunk and are controlled by the main or "sequence control" unit, or by the units which decode the commands.

Only two gates are activated at any one time, one gate which allows a selected register to transmit and another allowing a selected register to receive. Arithmetical functions are performed by the transmission of numbers into suitable gates connected to the "arithmetical organ" of the computer.

The main store and auxiliary store are connected to the "digit trunk", the store position being at any time under the control of a "store or memory control register". An "interpreter register" receives the commands one by one and decodes them and prepares the selected "function gates" for action. A "control or sequence register" is a counting register which stores at any time the store location of the next command to be selected. Its contents is normally increased by unity after the current command is selected.

Two registers, the "input and output registers" serve to make fresh data available to the machine and to provide results to the operator.

A block schematic diagram of the computer is given in fig. 6.

6. The operation sequence

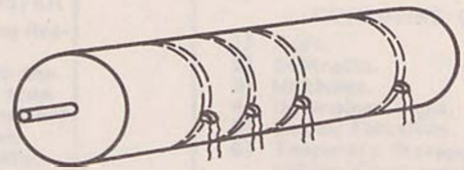
A regular routine of four basic operations is performed in an unvarying manner. First the contents of the "sequence register" is transferred via the digit trunk into the store or "memory control register" whose previous contents it replaces. The store is thereby prepared to read out the quantity held in the position denoted by the sequence control register.

As the selected position appears from the store its contents, that is the new command, is transferred via the digit trunk into the interpreter register where it is stored and decoded by suitable networks which prepare two function gates for action.

However the command may call for the store to transmit or receive, so the third act is to transmit the 10 digits of $p_{11} - p_{20}$ in the "interpreter register" out via the digit trunk into the "memory control register" whose contents it

DATA STORED AS MAGNETISED SPOTS ON SURFACE OF DRUM

1 20 Peripheral tracks.
2 1 'Clock' track with uniform spot train (1024 spots).
3 Numbers stored axially.
4 Capacity 1024, 20 digit numbers.
5 Access time 10 milliseconds.
6 Data read and recorded by same 'head'.



AUXILIARY EQUIPMENT

1 Position counter; counts angle of drum.
2 Position register; stores address from Computer.
3 Coincidence detector; impulsed when (1) & (2) are identical.
4 Input register } Connection to Computer.
5 Output register } Connection to Computer.
6 Gates, serial to parallel and parallel to serial.

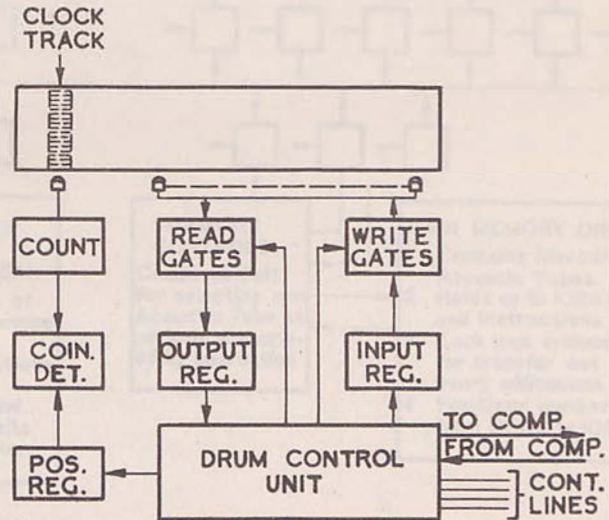


FIG. 5. MAGNETIC STORAGE.

1. All Numbers and Instructions are transferred via the Digit Trunk.
2. All Instructions are placed in the Main Memory Organ in serial order of performance.

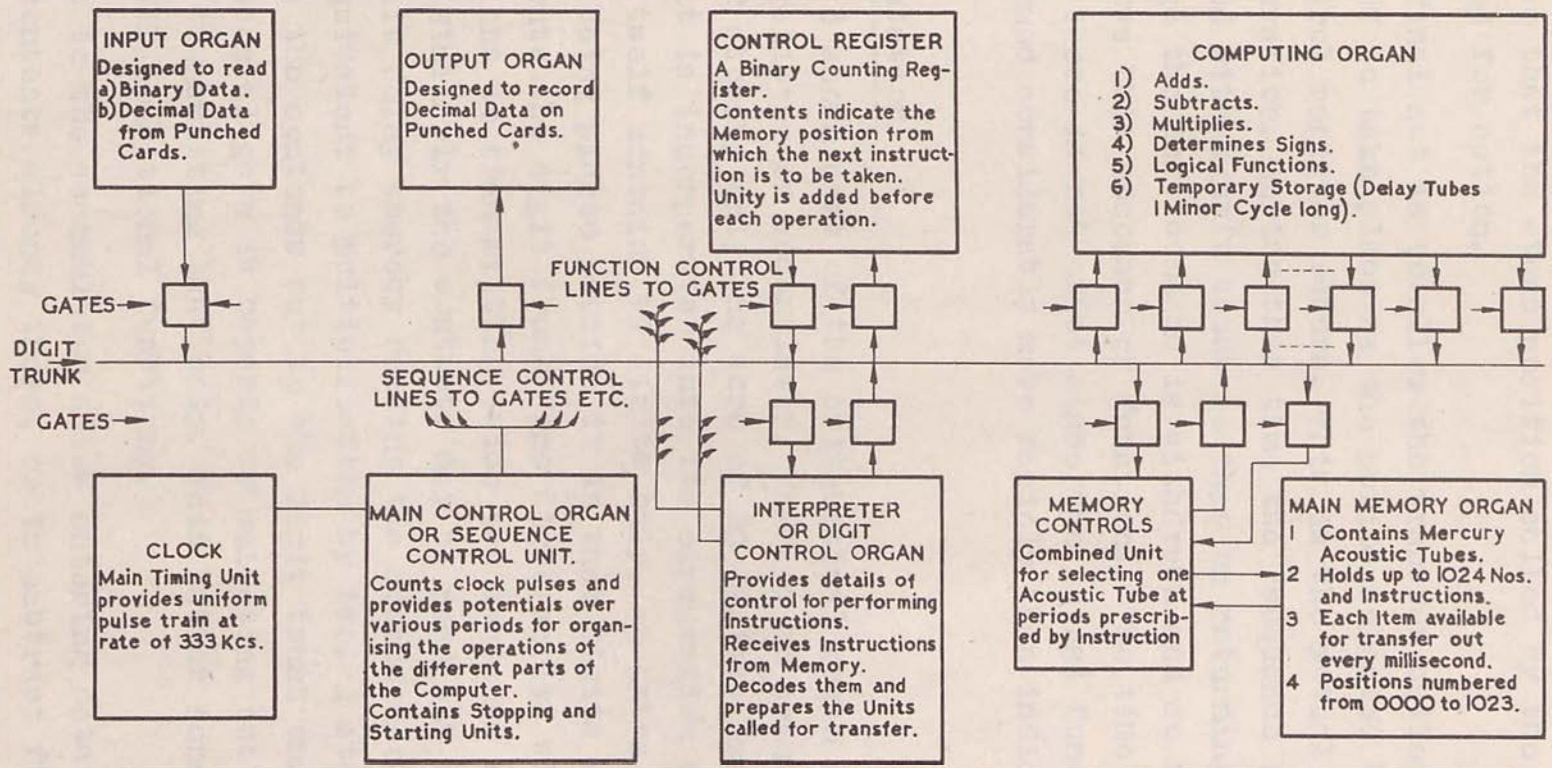


FIG. 6. BLOCK SCHEMATIC DIAGRAM OF COMPUTER.

replaces so that the store position called by the command is prepared for action.

The final act is to allow the transfer called for by the command to take place as the period selected by the store control register occurs. This is the actual computing operation. During this time the "sequence register" is provided with a unit count so that on returning to the first stage the next command is withdrawn and so on. Fig. 7 shows this sequence of functions. The time rate indicated there is not exact since the actual functions are performed considerably more rapidly than indicated.

7. Computation

Fig. 8 shows part of the arithmetical organ and shows a single register or accumulator. This is an acoustic tube capable of storing only one word of 20 digits, and one adding unit is incorporated into its circulation network. The tube itself contains 19 digits only, an extra delay of one digit being placed outside it so that digits may be read out onto the digit trunk "one digit early" which is equivalent to transmitting twice the contents of the register. Similarly the contents may be read out via a further unit delay thereby reading the contents out "late", equivalent to multiplication by two. A standard gate reads the contents out to the digit trunk unmodified whilst a special gate is capable of selecting out the sign digit and transmitting that only. This latter function is used for all conditional functions.

Gates to the accumulator allow entering data to add into the contents already there, or to subtract from it, or to substitute for it.

For instance, if we wish to double the quantity in the accumulator we could order the contents of this register to read out into the digit trunk and to travel along it and into the "add in" gate via which it would be added into itself. For clearing the accumulator we could instead order the contents out of the register and to enter the "subtract in" gate thereby subtracting from itself and resetting the register to zero.

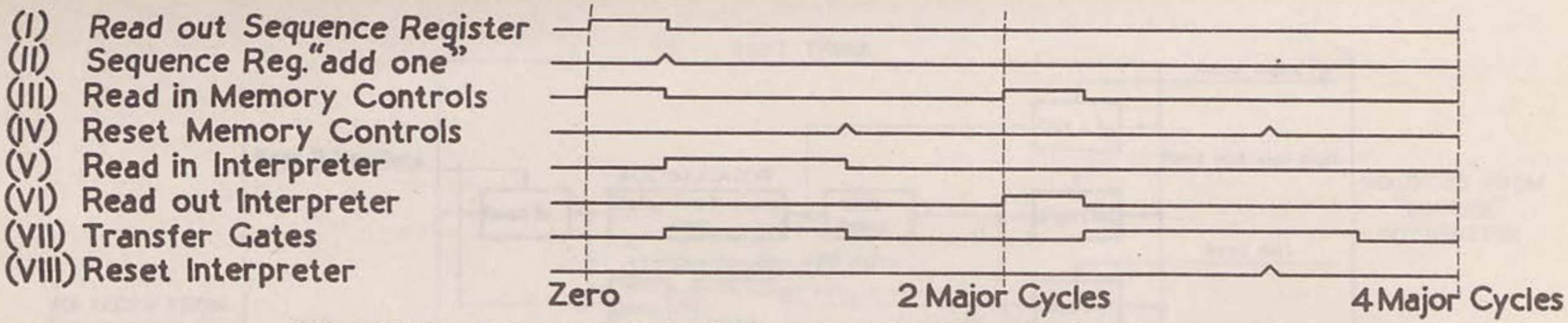
Other adding registers are incorporated in the arithmetical organ and a complete multiplying network is included.

8. <u>Input and output</u>

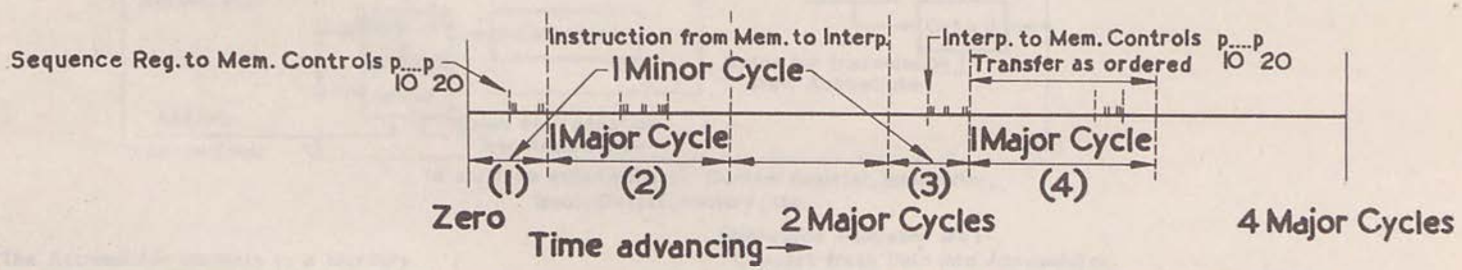
Commands and numbers are entered into the computer via punched cards. These cards are read in a columnar fashion by a row of twelve reading brushes and 20 digit words are compiled out of groups of 10 digit numbers from the cards. The assembly is organised by a special group of 20 commands entered initially into the computer via a group of uniselector switches. These commands are known as the "primary input".

Two means of recording results are provided; first, printed results are obtained via a teleprinter unit which is capable of printing decimal digit symbols and letters; and second, decimal data may be punched upon a Hollerith type card in a manner such that these may be listed on a standard card machine or may be reinserted into the card reader for use in a future programme. Fig. 9 shows the punched card input and output units.

VOLTAGE WAVEFORMS FOR CONTROL



TYPICAL SET OF PULSES APPEARING ON DIGIT TRUNK



SEQUENCE ROUTINE.

Sequence Reg. read to Memory Control Reg.

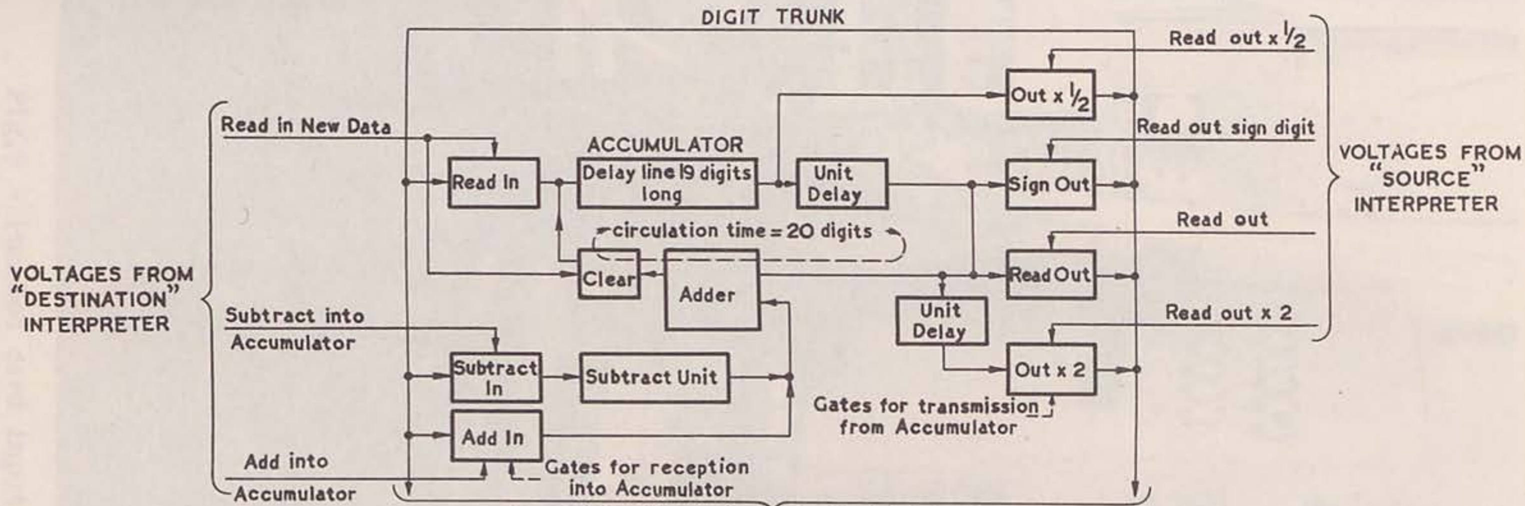
"Command" read from Memory to Interpreter.

Numerical Address of Command read to Memory Control Reg.

Transfer ordered by Command performed.

Return to (I).

FIG. 7. OPERATION SEQUENCE.



The Accumulator consists of a Mercury Acoustic Tube and Circulation Circuit similar to that of a Standard Memory Unit together with an Adding Unit.
Total Delay or capacity of circuit is 20 digits or 60 μ secs.

Operations illustrated are :-
1) Insert fresh Data into Accumulator
2) Add into Accumulator via "add gate"
3) Subtract from Accumulator via "subtract gate"
4) Transmit content of Accumulator
5) Transmit content of Accumulator multiplied by 2
6) Transmit content of Accumulator divided by 2
7) Transmit only the sign digit of content of Accumulator e.g. 1 in P_{20} position if negative, 0 if positive.

FIG. 8. COMPUTATION.
(PART OF ARITHMETIC NETWORK)

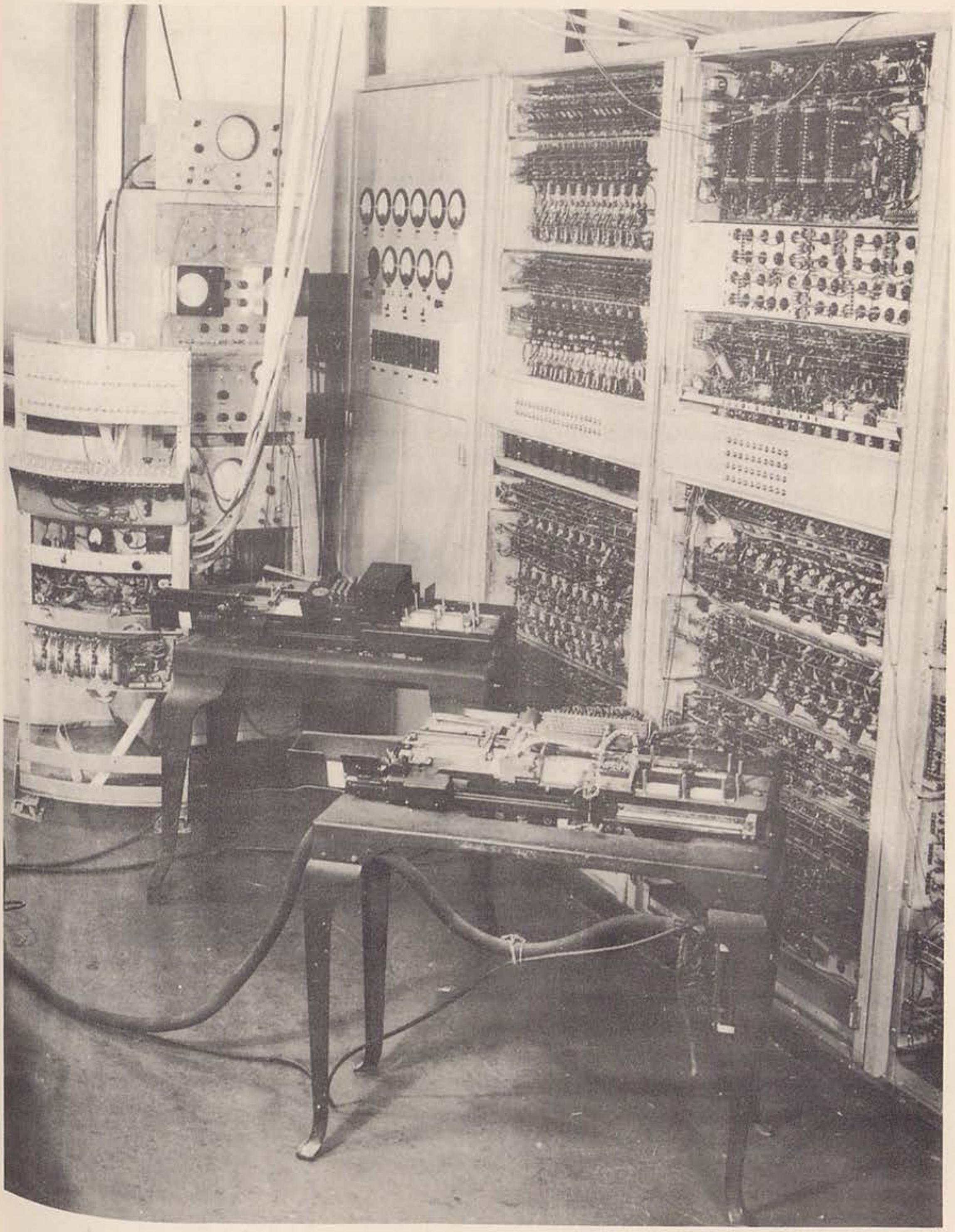


Fig. 9 - Punched card input and output units

9. Equipment

The general view of the equipment is shown in Fig.10. The leftmost large cabinet contains the acoustic store during assembly, the next to the right shows the arithmetical equipment, whilst the next shows the main control equipment. Fig. 11 shows a closer view of these cabinets in the top left of which the short acoustic tube arithmetical registers may be seen.

The final open cabinet in fig. 10 contains the input and output and pulse distribution equipment whilst the closed cabinet is the power supply. To the right of this, the auxiliary store is held.

A total of about 1500 tubes is included in the equipment, some of which may be eliminated at a later date as units of improved design are incorporated.

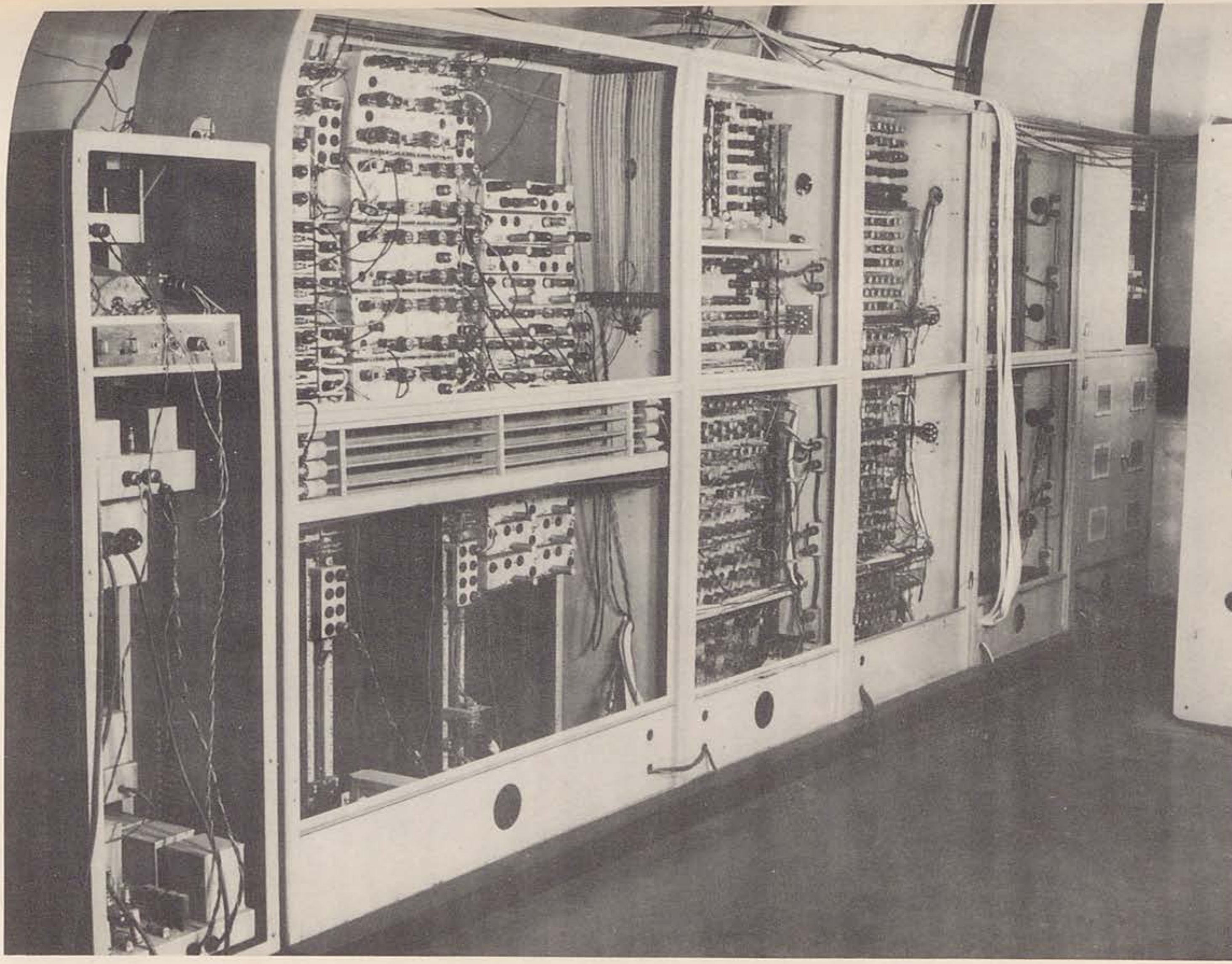


Fig.10 - General view of computer

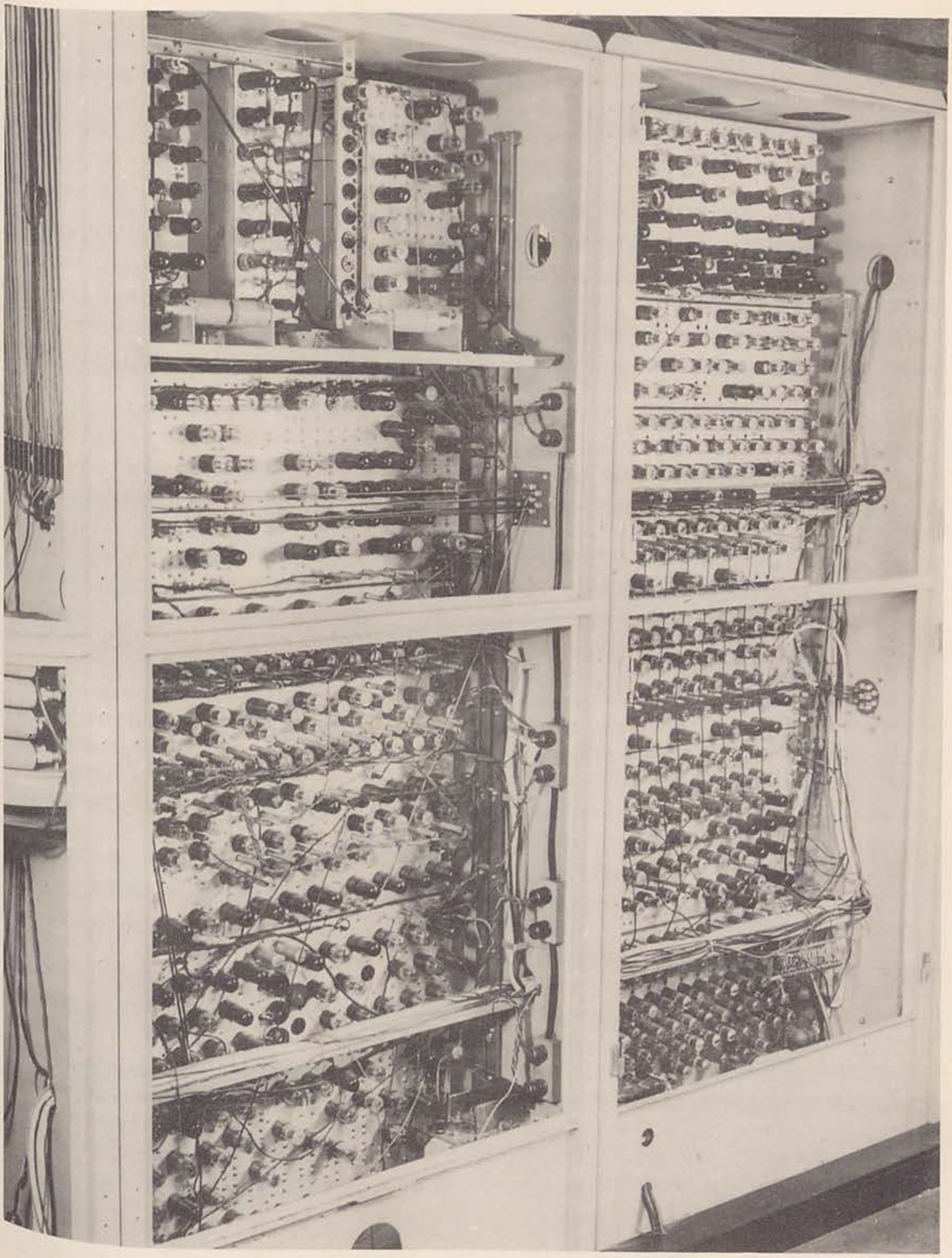


Fig. 11 - Main control equipment

Discussion on Papers I to IV

D. R. HARTREE - in reply to a question by Professor Astbury, dealt with methods of carrying out arithmetical operations, choosing the process of addition as an example. In adding two numbers a and b, three digits have to be added at each stage except the first. They are the digits of a and b and the carry digit c of the preceding stage. Professor Hartree outlined the principle of one device, due to von Neumann, for doing this (see ref. 20, chapter 8).

W. C. J. WHITE - drew attention to the use of ten digits in the C.S.I.R.O. digital computer for source and destination. This appeared to him to be an unnecessarily large number in view of the fact that the EDSAC used only seven.

T. PEARCEY - stated that the principal reason for doing this was to facilitate the adding of extra functional units if necessary by including extra chassis. This machine being an experimental one, the designers considered such a provision desirable. Coding was slightly complicated by the extra length of the code. Coding for source and destination enhances flexibility, and at the same time, the length of a number (20 digits) was made the same as that of a command, which simplifies the organization.

D. R. HARTREE - continuing discussion on this point, said that the source and destination together could be regarded as a specification of a single operation. It seemed unlikely that more than 2^6 or 2^7 operations would be required, so that to use ten digits to specify them wasted about three digital positions in each instruction, though this might not be a serious disadvantage. The policy of Cambridge had been to aim at simplicity of engineering possibly at the expense of programme length (see paper V, section 4(d)). Mr. Pearcey's remark about the C.S.I.R.O. machine being experimental should not be taken to suggest that the EDSAC and other machines were "production line" models. With the exception of the Ferranti machine, all others were experimental.

Professor Hartree extended his remarks when Mr. Stewart suggested that three extra binary digits be used for error-detecting codes. He considered that the use of error detecting procedures at each operation of the machine was a counsel of despair. Experience has shown that computers do not go wrong often enough to warrant this.

D. M. MYERS - said that one point had had little attention so far. Some compromise is necessary between hardware and programming. With regard to some processes it is necessary to decide whether they should be added to the library of sub-routines, or performed automatically by the computer. In the case of division, for example, one must decide whether to provide increased storage capacity to provide for instructions in a division subroutine, or new functional elements for dividing automatically.

In regard to programming and coding, it was encouraging to see new ideas being worked out for the sake of experience. In some years' time it would perhaps be best to have a standard approach so as to have a standard method of programming in different centres. Then an amicable compromise between programming and engineering would be necessary.

D. R. HARTREE - Mr. J. C. Stewart asked how boundary conditions are provided for on the differential analyser. Professor Hartree said that the main part of the answer is that the various lead screws of the integrators and plotting tables must be set to some appropriate position before the machine is started up. With one-point boundary conditions, this is straightforward as the dependent variable and its derivatives and any functions of them are known at the beginning of the range of the independent variable.

Difficulties arise however when some values of the variables are given at one point and the remainder at another, i.e. with two-point boundary conditions. For linear equations one could evaluate a particular integral and a sufficient number of complementary functions, all satisfying the boundary conditions at one end of the range, and forming a linear combination satisfying the conditions at the other end. His experience had been that this method is more theoretical than practical because of the sensitiveness of the solution to initial conditions. In such cases the use of trial solutions is preferable and for non-linear equations it is the only way. This is not as laborious as it sounds as no change in the general programme or set up of the machine is necessary but only a change in the setting of some of the integrators.

G. NEWSTEAD - referred to the equation solved by the differential analyser during the demonstration of its use and asked whether it was known beforehand if periodic solutions existed. Professor Hartree said no. The problem was proposed by Dr. MacLachlan who was interested in loud speaker performance. It was certain that there would be periodic solutions for small values of the coefficient of cos t. He wanted to know whether sub-harmonic solutions were possible.

G. E. BARLOW - described briefly a general purpose differential analyser at the Royal Aircraft Establishment.

It has been used chiefly for aircraft control and guided missile problems where the independent variable is real time. It has an overall accuracy of 1% - 1.5% and contains 15 integrators, 15 multipliers, 15 adding units and 8 cathode ray tube units.

D. M. MYERS - drew attention to the difficulty arising in most electrical integrating circuits, which integrate with respect to real time. Thus the argument of each integration must be the same and this adds greatly to the difficulty in putting many kinds of equations on the machine.

T. PEARCEY - said that the use of sinusoidal electrical quantities offered prospects for the solution of equations in a complex domain, particularly non-linear equations - an untouched field. It could also be used for inverse Laplace transforms with poles and singularities.

W. R. BLUNDEN - felt that as some of the delegates to this conference would not be attending further sessions some comment on the relative scope and usefulness of digital and analogue machines should be made at this stage.

The digital machine appeared to be superseding the analogue computing instrument mainly because of its greater versatility and accuracy.

The analogue class is limited by the degree of mechanical and electrical refinement of its construction. It is paradoxical that the ball and disc type integrator, which bridges the gap between the limit of a sum and a continuous integration and does in fact operate theoretically perfectly, gives an accuracy of say 1 in 1000, whereas a digital integrator using Simpson's Rule will evaluate an integral to any prescribed precision. This difference between the two methods illustrated above is fundamental and digital machines must be employed where high accuracy is necessary.

However, high accuracy is not always required and there are many cases where the accuracy of the data supplied to a machine does not warrant great precision in computing. Also much time may be saved by using certain classes of data given in graphical form in its analogue form rather than converting it to digital form.

From the point of view of versatility the digital machine has every advantage although it is worth remembering that the differential analyser has a very wide field of application by virtue of the great number of differential equations that arise in all branches of science and engineering. In this field it is universal. Also there are some special problems that require such frequent solution that a special purpose machine is necessary; for example, the anti-aircraft problem.

D. R. HARTREE - drew attention to two points about digital machines mentioned by Mr. Blunden - their versatility and accuracy. There is a third - their speed. In the case of predictors, he thought that the instantaneous results required might be given better by digital machines and also to greater accuracy.

Difficulties arise in using digital machines when the input is irregular or discontinuous. Analogue machines are convenient when the results of the calculation are needed in graphical form. However, it would be quite possible to develop some form of digital-analogue converter to produce a curve, on paper or on a cathode-ray tube, from data given in digital form.

If numbers are small, there is no need to waste storage capacity, for several numbers can be packed into one storage location (e.g. a 20-digit location can be divied into 4 groups each of 5 digits), it only needs sub-routines to

do the packing and unpacking. This is done with triple Fourier series for crystal structures where the input is only accurate to 1%.

Professor Hartree did not agree with the terminology in which the differential analyser is classed as a "general purpose" machine. It will solve differential equations, it will add and subtract, with a little ingenuity it can be used to multiply, with more it can be employed to solve simultaneous equations, but this is rather a sophisticated way of dealing with this particular calculation.

G. E. BARLOW - said that one advantage of analogue machines is that actual pieces of equipment can be put in and tested. That has been done at the Royal Aircraft Establishment.

T. PEARCEY - organic chemists require summations of triple Fourier series for electron distribution of a molecule. They give a vast table of data and require results in graphical form, which can only be done by hand. Often in the data the signs are not known and test summations have to be made to find the signs. A fast method should be available. Cathode-ray tube representation of the output of a digital machine could be used but analogue methods seem more desirable.

V. Introduction to Programming

by D. R. Hartree

I shall be concerned in this lecture not with automatic digital calculating machines themselves, but with the process of organising calculations for them, the process known as "programming". This will involve some ideas, and ways of thinking, which will probably be unfamiliar; but there will be nothing difficult in this introduction to the subject. It is possible to talk in general terms about programming, but I think it is much more enlightening to see a few examples worked in full; this will give some idea of the way one has to think about calculations in order to program them.

I have already emphasised (III, §6) that to the programmer the most important things about a machine are not what is the physical form of the store or in what form numbers are represented in the machine, but first, what is the standard form for instructions, secondly, what are the standard instructions for single processes, and thirdly how does the machine, having carried out the operation specified by one instruction, determine the next instruction?

In order to show some illustrative examples, I shall have to work in terms of a particular form of instructions, and a particular way of writing them, and of determining the next instruction; but I do not think this matters because once the ideas involved in one system of programming have been grasped, it will be relatively easy to change over to another system, provided it is not too complicated. I shall present the subject in terms of the system of programming used on the EDSAC, the machine at the Mathematical Laboratory at the University of Cambridge, both because this is the system with which I am most familiar, and because it seems to me the most simple and straight-forward that I know. I present it not as the best system, but as one which has been tested by experience and practical application to calculations of various kinds over a period of two years. It has proved simple enough for workers in other laboratories without previous acquaintance with the subject, to learn in two or three weeks enough to program their own calculations. And it illustrates some of the facilities which must be provided in some way by any system of programming which is likely to be of practical value to the user.

2. Coding of instructions for the EDSAC.

In this machine, instructions are represented in the same form as numbers and are stored in the same store; instructions are of the one-address type stored serially (see III, §4), that is, instructions are normally stored at addresses numbered in the sequence in which they are to be carried out.

Instructions are coded as follows. An instruction, like a number, is represented in the machine by an ordered set of digits, each of which may be 0 or 1:-

```
x x x x x x x x x . . . . . . . . x x x
 Operation |        Address        |
           Spare                   Terminal
                                   digit
```

The digits in the five most significant digital positions (including that which, in a number, is used for sign indication) specify, in coded form, an operation to be carried out, such as an addition, or a transfer of a number from the arithmetical unit to an address in the store. The next digital position is a spare, and the next ten specify, in most instructions, the address in the store to which this operation refers. Then there is a terminal digit with which I shall not be concered; it will be zero in all the examples which I shall use.

One needs a form in which to write such an instruction; the form adopted is a letter representing the operation (A for add, S for subtract, T for transfer from the arithmetical unit to the store, etc.) followed by the address written in decimal form; for example, the instruction written A 38 means "add the content of storage location 38 into the accumulator". This written form is closely related to the form in which the instructions are punched on the input tape. The following are the written forms of the kinds of instructions used in the examples I shall consider:-

A n	Add C(n) into Acc.
S n	Add -C(n) into Acc.
T n	Transfer C(Acc) to n and clear Acc.
U n	Transfer C(Acc) to n and hold in Acc.
E n	Examine C(Acc); select the next instruction as follows:-

if C(Acc) $\geq$ 0, take C(n) as the next instruction
if C(Acc) $<$ 0, proceed serially; that is, if the instruction E n is in address k, take C(k + 1) as the next instruction.

G n   Examine C(Acc); select the next instruction as follows:-

If $C(Acc) < 0$, take C(n) as the next instruction.

If $C(Acc) \geq 0$, proceed serially.

Z   Stop.

The only way in which the content of storage location n can be altered is by an instruction T n or U n, which replaces the previous content of this storage location by the current content of the accumulator. The letter P is represented by the digits 00000, so that the number n, in terms of the units digit of the address in an instruction, can be written, in the form of an instruction, as P n.

These are not all the kinds of instructions used in this machine, but they are the only ones I shall need to use; for the full set, see references 56, 57.

3. First example

Now consider, as a first very simple example, the set of instructions to carry out the calculation

$$|C(6)| \text{ to } 4,$$

this being considered as a part of a larger calculation, and these instructions being entered with the accumulator clear. There are several ways in which this could be done, and only one will be considered here. Let us write x for C(6) for short.

We want the machine to carry out a different set of operations according as x is positive or negative. It can only discriminate on the sign of a number when that number is the content of the accumulator, so the first step is to place x in the accumulator by the instruction A 6. Let this instruction be located at address m in the store, so that we have:-

Address	Content	C(Acc)
m	A 6	x.

C(Acc) here is the content of the accumulator after the instruction has been carried out.

The next instruction will be taken from address (m + 1), and let us use an instruction of the type E n to discriminate on the sign of x:-

m + 1	E n	x.

Let us first consider what happens if x is negative, and leave the value of n, from which the next instruction is taken if x is positive, unspecified for the moment. Since in this case C(Acc) = x is negative, the next instruction is taken from address m + 2. In this case, $|x|$ is -x, so what we want the machine to do is to replace x in the accumulator by -x, which can then be sent to address 4 by the instruction T 4. This can be done in various ways; one way is to use the two instructions T q, which sends x to address q and clears the accumulator, followed by S q which places -x in the accumulator; it is often convenient to use q = 0 as a temporary address for numbers which are going to be used again immediately. Thus we have the instructions:-

m + 2	T 0	0	C(0) = x
m + 3	S 0	−x	
m + 4	T 4	0,	

and these complete the calculation for the case when x is negative.

Now consider what happens when x is positive. In this case the content of the accumulator when the instruction E n comes to be carried out is positive and is already the quantity $|x|$ which we want to send to address 4, that is to say, when x is positive we want to follow the instruction E n directly by the instruction T 4. But we already have the instruction T 4 in address m + 4, so that we can obtain the result required by giving n here the value m + 4; so that we finally reach the group of instructions:-

Address	Content
m	A 6
m + 1	E (m + 4)
m + 2	T 0
m + 3	S 0
m + 4	T 4

I have considered the argument here in detail, as it will be unfamiliar in character to some. It is not difficult but it does involve an unfamiliar way of thinking and a clear head, and some practice is needed before one can program more complicated procedures fluently and correctly.

If every calculation had to be programmed from scratch in as much detail as this, programming would be a long and sometimes laborious process, but as I have already explained, (III, §6), it can be lightened very considerably by the use of a library of sub-routines for standard processes, which can be programmed and thoroughly checked once for all. This group of operations is too simple to be incorporated as a library sub-routine, but let us consider it as such, as it illustrates in a simple form some of the general points which arise in connection with the formation and use of a library of sub-routines. An objection has been made to the whole idea of using such a library, that it would be too inflexible; unless it were very large, one would find in practice that one usually wanted not exactly the library form of a sub-routine but some variant of it. The answer to this, I think, is that the sub-routines should be drawn up in a form which <u>does</u> provide the required degree of flexibility, and that this can in fact be done.

These instructions are not suitable for incorporation into a library as they stand, because the address in one of them, the instruction E (m + 4), depends on the position in the store in which these instructions themselves are placed, and to cover all possibilities we would either have to restrict ourselves to putting this sub-routine always in a definite position or in one of a small number of definite positions in the store, which would be an unacceptable restriction in a machine without an auxiliary store, or we would have to have in the library a whole lot of such groups of instructions, one for each possible value of m. The first step in making these sub-routines flexible enough is to draw them up in a form which is independent of the value which m may have in any particular use of them.

This can be done in various ways; with the EDSAC it is done by having the instructions <u>punched on the input tape</u> in a form rather different than that in which they appear in the store, and making the translation from one form to the other in the course of reading the tape. The instruction already worked out as an example would be punched as follows:-

Address	Content	Entry on Tape
		G K
m	A 6	A 6 F
m + 1	E (m + 4)	E 4 θ
m + 2	T 0	T F
m + 3	S 0	S F
m + 4	T 4	T 4 F
m + 5	Next instruction after calculation C(6) to 4 completed.	

Each instruction is terminated by a code letter, which happens to be F if the numeral in the instruction is to be read as it stands as the address to which that instruction refers (address 0 does not have to be punched explicitly), and θ if the value of m has to be added to the numeral in the address as punched, in order to give the address to which the instruction refers. Another way of putting this is that the terminal code letter θ indicates that the numeral represents a relative address, that is, an address relative to the address of the first of this group of instructions, whereas F indicates that the numeral represents an absolute address. In order to specify the value of m to be used in any particular application of these instructions, they are preceded on the tape by the code letters G K, which form an indication to the machine that in the process of reading the tape the address into which the next instruction is placed is to be recorded, for use in reading instructions terminated by the code letter θ.

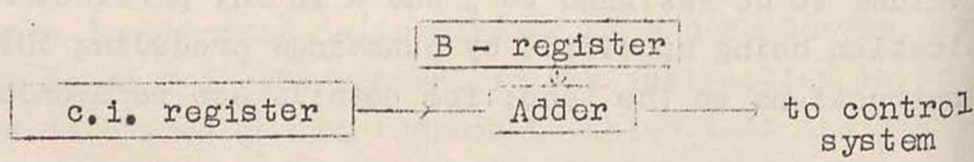
This use of terminal code letters has put this group of instructions in a form which is independent of where in the store the instructions themselves are placed. This way of making sub-routines more flexible can be carried a good deal further. This group of instructions, regarded as a sub-routine, is still too specific to be suitable for incorporation in a library, in that it refers explicitly to the address 6 of the number whose modulus is to be found and the address 4 where the result is to be placed. By an extension of the use of terminal code letters, these instructions can be punched in such a form that they can be used for any calculation of the type

$$|\, C(j) \,| \text{ to } k,$$

the values to be assigned to j and k in any particular application being specified by punchings preceding this group of instructions on the tape; for details see references 52, 57.

The code letters G K, and the terminal code letters F, θ, and others, do not appear in the instructions in the store; they are indications to the machine regarding the way in which tape entries are to be translated into contents of storage locations. Some machine operation is required at this stage, in order to convert the decimal form in which addresses are punched on the tape into the binary form in which they are represented in the store, and the further operations involved in using the terminal code letters can be included at this stage. The set of instructions for enabling the machine to read the tape and do these operations on tape entries (52) can be worked out once for all, and the facilities it provides can then be used, in most cases, without knowing how in fact the machine does carry out these operations on the tape entries. Some machines require that the addresses in their instructions should be represented to the machine in a special way, in scale of 2 or 8 or 32 for example; it is worth putting on record that at Cambridge there has never been any reason to consider presenting them in other than their decimal form.

The facilities which are provided in the EDSAC by terminal code letters in the punched instructions, which are interpreted in the course of reading the tape, can be provided in other ways. One way is to include in the connection between the current instruction register (III, §5) and the control unit an adder, by which the content of some other register (B - register) can be added



to the instruction in the current instruction register (see III, fig. 1) as it is passed to the control system; a method of this kind is used in the Ferranti machine at the University of Manchester, though in this machine the B-register has other functions as well.

4. Second example

I have mentioned that in most of the more recent machines numbers and instructions are represented in the machine in the same form, and that the most important result of this is that it provides the possibility of the machine automatically modifying its own operating instructions in the course of a calculation. The following provides a simple example of the use of this facility. Suppose we want the machine to do the calculation:- Add the contents of storage location 100 to 140 inclusive and send the result to location 170, or in symbols

$$\sum_{j=100}^{140} C(j) \text{ to } 170; \qquad (1)$$

this being one component of a larger calculation.

The simplest, and quickest, way of doing this is by means of the sequence of instructions (entered with the accumulator clear):-

A 100, A 101, A 102, ..., A 139, A 140, T 170. (2)

But this is extravagant of storage space for instructions, and would be still more so if the number of numbers to be added were 100 or 200 instead of 41. Another way is to use repeatedly the sequence of instructions (entered with the accumulator clear) A 170, A n, T 170, which result in the content of storage location 170 being increased by the content of storage location n, with n taking the values 100, 101, 102 ... successively. Suppose these instructions are at addresses (m + 1), (m + 2), (m + 3), address m being reserved for a preliminary instruction as we shall see later:-

Address	Content
m	
m + 1	A 170
m + 2	A n
m + 3	T 170

Then what we want the machine to do is to take the content of address (m + 2) and add 1 to it, in the units place of the digits of n. Unity in this place (which, as explained at the end of §2, is represented in the form of an instruction by P 1) is so often wanted in just this kind of context that it is kept as a permanent constant content of address 2:-

2 P 1

and the required addition of unity to the address in the instruction A n is achieved by the three instructions

Address	Content	C(Acc)
m + 4	A (m + 2)	A n
m + 5	A 2	A (n + 1)
m + 6	T (m + 2)	0

Now the machine would be ready to repeat the instructions in addresses (m + 1) to (m + 6) with the new value of n, except for this, that it has not been provided with any means of determining when to stop repeating this sequence of operations. There are several ways of providing this. One method is to use one storage location as a counter, whose content is increased by unity for each repetition of this group of instructions and is tested each time to see if the count is complete. But this is not necessary; the variable instruction in address (m + 2) can itself be used as a counter. The following is not the most elegant way of doing this, but is probably the easiest to follow.

When the addition is complete, the instruction A (n + 1) newly formed in the accumulator as a result of the instruction in address (m + 5), will be A 141, whereas if the addition is incomplete, the address in it will be less than 141. If we subtract from it the "word" which, written as an instruction, is A 141, the result will be n-140, where n is the address of the last number to be added; this is zero if the addition is complete and negative if it is incomplete, and this can be used as the basis for a discrimination.

Let us therefore replace the instruction in address (m + 6) by U (m + 2) so as to retain A (n + 1) in the accumulator, and subtract A 141, which we will suppose stored in address p:-

Address	Content	C(Acc)
p	A 141	
m + 6	U (m + 2)	A (n + 1)
m + 7	S p	n-140

Now let us discriminate using an E instruction:-

m + 8	E q,

and consider first what happens if the addition is incomplete;

then C(Acc) = (n-140) is negative and the next instruction is taken from location (m + 8). In this case we want to repeat the addition process by returning to the instructions in (m + 2) with the accumulator clear. We therefore clear the accumulator:-

m + 9   T 0,

and, the content of the accumulator being now zero, the instruction

m + 10   E (m + 1)

will now result in the next instruction being taken from address (m + 1) to repeat the addition process with the new address in the instruction in address (m + 2). A convenient address for the "word" A 141, used to test for the completion of the addition, is immediately after the last of these instructions; if this address is chosen, this fixes the value of p as (m + 11), and the next instruction after the addition is complete can then be in address (m + 12). If the addition is complete, the content of the accumulator is zero when the instruction in address (m + 8) comes to be carried out, so the address in this instruction must be (m + 12).

These instructions must be entered with address 170 clear, otherwise the initial content of address 170 would be added into the sum we want the machine to calculate. Assuming the accumulator to be left clear as a result of the immediately preceding instruction, this can be done by the preliminary instruction T 170. Hence we reach the following group of instructions, written here both in the form they take in the store and in the form they would be punched on the tape so as to be formally independent of their location in the store:-

Address	Content	Entry on Tape	Notes
		G K	
m	T 170	T 170 F	Preliminary clearing operation
m + 1	A 170	A 170 F	The addition process
m + 2	A 100	A 100 F	
m + 3	T 170	T 170 F	
m + 4	A (m + 2)	A 2 θ	Advance address in instruction in (m + 1)
m + 5	A 2	A 2 F	
m + 6	U (m + 2)	U 2 θ	
m + 7	S (m + 11)	S 11 θ	Test if addition complete
m + 8	E (m + 12)	E 12 θ	
m + 9	T 0	T F	Return to instruction in (m + 1) with accumulator clear if addition not complete
m + 10	E (m + 1)	E 1 θ	
m + 11	A 141	A 141 F	
m + 12	Next instruction after addition complete		

This group of instructions illustrates several general points:-

(a) Only the three instructions in address (m + 1), (m + 2), (m + 3) are actually concerned with carrying out the arithmetic of the addition process with which the calculation is concerned; the rest are concerned with organising how the calculation shall be done rather than in doing it. This is characteristic of programming for machines which provide facilities for carrying out operations on their own operating instructions during a calculation, and is also one of the most interesting features of this programming. Programming a sequence of purely arithmetical operations is usually fairly straightforward and rather dull; programming becomes really interesting when it becomes concerned with the more organisational features of a calculation.

(b) A set of the same twelve instructions of this form can be used to form the sum of a group of numbers in successive addresses, however many numbers have to be added. Thus the space required for instructions does not necessarily increase with the scale of the calculation. This again is characteristic of programming for machines which provide facilities for making the machine modify its own operating instructions.

(c) A group of ten instructions *, from that in address (m + 1) to that in address (m + 10) has to be carried out for each number to be added in forming the sum (1). Thus this process is about ten times as slow as the process represented by the set of instructions (2). Unless individual operations could be carried out fast, we might not consider that we could afford the time for this slower process. This illustrates one advantage of the high speed of individual operations which the use of electronic circuits makes possible. This high speed is important not so much for the high overall speed in long calculations, as for what one can buy with it. One thing that can be bought with it is economy of storage for instructions, as mentioned under (b) above; another thing one can buy is simplicity of engineering, for if the machine carries out individual operations fast enough, one may be able to afford the time to carry out, by means of programmed sub-routines, operations such as division, square root, decimal-binary and binary-decimal conversions, for which one would otherwise build special pieces of hardware.

* By use of other methods for counting and testing, the number of instructions in the repetitive group can be reduced to seven or eight.

(d) It is necessary to transfer out of the accumulator the partial sum formed at each stage, in order to have the accumulator free for other operations such as modifying the instruction in address (m + 1) and testing whether the addition is complete. This could be avoided if there were two accumulators, one for accumulating the sum to be calculated, and the other for modifying instructions and for the testing process. But this would only suffice in a few cases; for less simple calculations we would feel the need for a third accumulator for other intermediate results. Programming could certainly be shortened, though I am not sure that it would be simplified, by duplication of machine components in this way, and conversely the engineering can be simplified by putting more on to the programming. Here there seems to be a difference of policy between the group at the Mathematical Laboratory at Cambridge and the group at C.S.I.R.O. Radiophysics in Sydney. The policy at Cambridge is to simplify the engineering at the cost of lengthening the programming; one of the ideas behind this is that much of the programming will be incorporated into library subroutines which can be worked out and checked once for all, and then need no testing or maintenance, whereas hardware needs both. The policy at Sydney is to shorten the program by incorporating more components in the machine.

(e) In this example as programmed, the address of the first addend, which is the initial address in the instruction in (m + 2), is lost. If this set of instructions has to be repeated, this address would have to be replaced, or better, inserted by some further preliminary instructions. But there is another, more elegant way of carrying out the count of the number of repetitions of the addition process *, in which the

* See ref. 57, appendix E on counting.

count and the test for the completion of the process are combined, and into which the setting of the address of the first addend can be incorporated.

(f) The "word" A 141 in address (m + 11) is never actually carried out as an instruction, it is only incorporated as a standard with which to compare the instruction A (n + 1) manufactured as a result of the instruction in address (m + 5). The number in the sequence control register takes the successive values:-

... (m + 6), (m + 7), (m + 8), (m + 9), (m + 10), (m + 2), ... if the addition is incomplete, or:-

... (m + 6), (m + 7), (m + 8), (m + 12), ...

if the addition is complete; there is no way in which it can ever take the value (m + 11) which would result in the instruction A 141 being carried out.

(g) The relations between the form of the instruction as punched on the tape and as contents of storage locations are quite systematic, so the machine itself can be used to effect the translation from the tape form to the store form, this is done automatically in the process of reading the tape. The space required for the instructions for this purpose can be used for other purposes once the instructions have been read from the tape.

5. Entering and leaving sub-routines.

The above group of instructions for the evaluation of the sum (1) has been thought of as preceded by an instruction in address (m - 1) immediately preceding them, and followed by an instruction in address (m + 12) immediately succeeding them. But this would not be convenient if this set of instructions had to be used several times, with different points of leaving and re-entering the main program. For this purpose we want to draw up sub-routines in such a form that they can be placed in the store quite independently of the main program. This again can be done in several ways. I shall outline the method used at Cambridge (52), as it provides another example of the process of using the machine to modify its own operating instructions, in this case by altering the operation specified as well as the address.

The instruction for entering the sub-routine is sometimes known as the "cue" and that for leaving the sub-routine and returning to the main program as the "link". Consider the instructions for the process

$|C(6)|$ to 4

treated as a closed sub-routine. This is programmed as follows:

Address	Content	Entry on Tape	C(Acc)	Notes
3	U 2	U 2 F		U = E-A

Main program

The following instructions are entered with the accumulator clear:-

m	A m		A m	
m + 1	G n		A m	Cue
m + 2	Next instruction after process covered by the sub-routine			

Sub-routine

n			A m	On entry into sub-routine
	A 3	A 3 F	E (m + 2)	Form Link

n + 1	T (n + 7)	T 7 θ	0	Plant link
n + 2	A 6	A 6 F		
n + 3	E (n + 6)	E 6 θ	(0)(6)	
n + 4	T 0	T F	0	
n + 5	S 0	S F	-0 (6)	
n + 6	T 4	T 4 F	0	
n + 7	Z	Z F		Becomes link instruction E (m + 2) as result of instruction in (n + 1).

The instruction A m in address m adds itself into the accumulator; the group of digits represented by A has a 1 in the most significant digital position, which is the sign-digital position if the "word" A m is regarded as a number. Hence the instruction G n results in the next instruction being taken from address n, the beginning of the sub-routine. Thus the sub-routine is entered with A m in the accumulator. The link instruction for return to the main program is E (m + 2), and this can be constructed by adding to the accumulator the group of digits which is represented as an instruction by U 2, and which is kept as a permanent content of storage location 3 for this purpose. The next instruction of the sub-routine plants the link instruction, just formed, at the end of the sub-routine, which in this case is address (n + 7). Then follow the instructions for the process 0(6) to 4, of which the last clears the accumulator, and when these are completed the link instruction, previously planted, results in the next instruction being taken from address (m + 2), with the accumulator clear. The instructions for forming and planting the link instruction are part of the sub-routine, and all the programmer needs to do is to provide the two instructions in the main program for entering the sub-routine.

The original content of storage location (n + 7) here is irrelevant to the program, as it is replaced by the link instruction before C(n + 7) is used as an instruction. The stop instruction Z is sometimes used, as here, for the initial content of such a storage location, so that if the link instruction is incorrectly planted, through a fault either of programming or of machine operation, the machine will stop with the sequence control register holding the address at which it should have been planted, and this will give a clue to the fault.

6. Interpretive sub-routines.

I have already mentioned the organisational character of much of the programming for a machine provided with facilities for modifying its own operating instructions. A development of this, on which a good deal of work has been done at Cambridge in the last year, is concerned with what have come to be called "interpretive" sub-routines, because they instruct the machine about special interpretations to be given to its operating instructions. For example, one can set aside two storage locations, say 6 and 7, to form the "real" and "imaginary" parts of an accumulator for complex numbers, and then provide the machine with a set of instructions to interpret the single instruction A p as meaning "add the contents of p and (p + 1), regarded as the real and imaginary parts of a complex number, into the complex accumulator" and similarly for other operations on complex numbers. For addition, this means expanding the instructions A p into the group of six instructions:-

A 6, A p, T 6, A 7, A (p + 1), T 7

instead of carrying it out directly, and this operation of "processing" the instructions before carrying them out is the kind of thing which the machine does in following an interpretive sub-routine.

Other interpretive sub-routines have been drawn up to deal with numbers expressed in floating-decimal form, and in coded decimal form, and for operations on double length numbers.

In conclusion, it should be recorded that a book (57) on programming containing copies of all the important sub-routines in the library associated with the EDSKE at Cambridge up to the beginning of 1951 has recently been published; this forms a very considerable store of knowledge and experience in this aspect of the use of automatic digital calculating machines.

Discussion

T. G. ROOM - asked why it is necessary to specify the transfer instruction T (n + 7) in the example in section 5.

D. R. HARTREE - pointed out that the only way to alter the contents of a storage register is to put new data into it. With systematic programming this presents no difficulty.

In reply to questions by Mr. Stewart on the possibility of saving machine time by reducing the number of steps in the sub-routine for addition, Professor Hartree said that by the process given in section 4, the machine would take about two thirds of a second to add 40 numbers; to work out a less straightforward routine, which however would be quicker for the machine, would take much longer than this. It is often more economical to take advantage of the machine's speed by using simple procedures which are easy to program rather than elaborate routines which might save second or minutes of machine time but take minutes or hours more to program. Mr. Stewart then asked whether it would be necessary in adding (say) 500 numbers to hold them all in the store. Replying, Professor Hartree said that if the data, instructions and working space were within the capacity of the machine, it was the usual practice to read in all the data and instructions before the calculation was started. However, when this was not possible it might be possible to arrange the calculation so that some of the data or instructions, or both, could be read in and used and other data or instructions read in later. For example, in calculations such as the evaluation of sums x^2, y^2, z^2, ... xy, xz, yz, ..., the data could be divided into blocks, each block consisting of one set of values of x, y, z, ... The data in one block could be read in from the tape, operations carried out on these data, and other blocks subsequently treated in the same way. It is possible to print out the results for one block before reading in data for the next.

In replying to a question by Professor Cherry, Professor Hartree explained the method of specifying and carrying out multiplication. He said two steps are required - one to put the multiplier into the multiplier register and the second to carry out the multiplication. The instructions used are:-

H n	Transfer C(n) into Multiplier Register
V n	Multiply C(n) by C(M.R.) and add into Accumulator
N n	Multiply -C(n) by C(M.R.) and add into Accumulator

Professor Hartree added that other arithmetic functions are right shift, left shift, round off, so called "logical multiplication" (also called "collation" and "conjunction"). Logical multiplication is useful, for by this means some of the digits of a number (the first five for example) can be separated from the rest.

In answering a question by Mr. Blunden on the use of the term "accumulator", Professor Hartree said that in early machines there was no sharp distinction between the accumulator used as a storage unit and as an adding unit. At Cambridge the accumulator is in the arithmetic unit.

In concluding the discussion on this paper, Professor Hartree pointed out that complex numbers could be dealt with by using one accumulator and two special storage positions and said that a full account of this is given by Wilkes, Wheeler and Gill (57).

VI. Programming for the C.S.I.R.O. Digital Machine

by T. Pearcey

Professor Hartree (V) has discussed the principles of programming procedures with particular relation to the Cambridge machine, EDSAC. That these principles are fundamental to the design of programmes is evidenced by their appearance in programming processes for other machines, in particular for the Mk. I Computer in the Division of Radiophysics.

Certain differences between the approach to programming for this machine and that for the EDSAC are due to the different methods adopted for coding commands in the two machines.

2. The Command Code

The command system adopted for the Mk. I Computer is of a modified two-address type, where two registers are specified for the performance of an operation. All operations are considered as transfers of numbers from any one register, as "source", to any other one register, as "destination". Specification of the source and destination occupies two addresses. The main store is considered as a single address, and a third sub-address is included to specify which position in the store is to be adopted if the store is called for as either source or destination.

A command is transmitted serially and its digits appear on the digit trunk in a prescribed order, reading from the first digit denoted by p_1, through p_2, p_3 etc. to p_{20}, the last digit. Each of these digits may independently be given the values 1 or 0.

The following groups comprise the addresses:-

p_1 to p_5 specify up to 32 destinations

p_6 to p_{10} " " " 32 sources

p_{11} to p_{20} " " " 1024 sub-address registers of the store

Commands constituting a full programme are held within the main store and are adopted for use in a sequential manner unless a command specifies otherwise.

3. Numerical Code

The numerical code adopted is the straight binary scale, with negative numbers recorded as complements. A special convention is adopted in the case of multiplication, that the binary point be between positions p_{19} and p_{20}. If α_j is the coefficient of the digit p_j, then p_j is given the weighting 2^{+j-20}, and a number is recorded as

$$N = \sum_{j=1}^{19} \alpha_j 2^{j-20}$$

$$\alpha_j = 0 \text{ or } 1; \; j = 1, 2, \text{ -- } 19$$

$$\text{and } -N = 2 - \sum_{j=1}^{19} \alpha_j 2^{j-20}$$

The digit p_{20} therefore occupies the place of a sign-digit, 0 if positive, 1 if negative.

4. Registers and Function Gates

To each of a group of computing registers there are attached a number of "function gates". These gates are specified directly by the code of the command and subject the incoming or outgoing information to numerical or logical transformation during transfer.

The main computing registers are denoted by the letters - A, B, C, D and H.

Register A is the main accumulator and possesses, among others, the function gates by which numbers may be added in, subtracted in or substituted into the register; and may read out and hold, read out and reset, and read out the sign digit of the contents. These facilities are also possessed by registers C and D except that of "read out and reset".

Register B is a single storage register which can also be used to hold a multiplier, whilst C holds a multiplicand. B can also read out its sign digit.

Register D can hold up to 16 numbers or words, the position being denoted by suffices, D_j say. Register H is a 10 digit register capable of reading and substituting in the digit groups p_1 to p_{10}, or p_{11} to p_{20}, and reading out in either of the same groups.

Serially positioned commands are specified by a "sequence register", S. This is a 10 digit register operating in positions p_{11} to p_{20} only. It receives a unit which is counted into its contents prior to its use for selecting a command. It is also capable of addition and substitution.

An interpreter register receives the commands, and is capable of reading out its contents in positions p_{11} to p_{20} only.

5. Notation

A transfer is denoted by an arrow notation connected with the letter symbols denoting the transmitting and receiving registers. The contents of a register is indicated by a bracket notation surrounding the register symbol, and functions during transfer are indicated by arithmetical symbols associated with the appropriate registers.

An unbracketed number to the left of an arrow implies that that number is written into the $p_{11} - p_{20}$ address position and that the interpreter register is the source. Numbers in brackets denote the contents of the corresponding store position.

The symbols H_l and H_u are used to denote the "lower group" $p_1 - p_{10}$, or "upper group" $p_{11} - p_{20}$ functions of the H register.

For example:-

$(53) \longrightarrow A$	implies clear register A and read to it the contents of store 53
$(53) \xrightarrow{+} A$	implies add into A the contents of store 53
$53 \longrightarrow A$	implies clear A and read the number 53 p_{11} into it from the interpreter

6. Construction of Constants within Sub-routines

A frequent need is the use of special constants applicable to a current sub-routine. A number $N2^{-9} + M2^{-19}$, where N and M are integers less than 2^{10} can be compiled by means of the H register and the interpreter as follows:-

$$M \longrightarrow H_u$$

$$N \longrightarrow A$$

$$(H_1) \xrightarrow{+} A$$

Thus "M" is read into a cleared A register by the second command, whilst "N" is read into the H register by the first and is in effect shifted to the $p_1 - p_{10}$ position by the third command which adds $M2^{-19}$ on to $N2^{-9}$ which is already in A.

Frequently required constants are the $p_{20} - p_{17}$ group of digits.

7. Break of Sequential Commands

The sequence register possesses the facility of addition, substitution and counting.

A command may call for an immediate break in the sequential routine by substitution of a fresh number into the sequence register. Thus the command

$n \longrightarrow S$ implies reset register S and substitute "n" from the interpreter

The next command is obtained from position "n" in the store.

However a relative shift of the sequential routine may be made either forward or backward by the command

$m \xrightarrow{+} S$ implying add to the contents of S the number "m" from the interpreter

This advances "m" positions forward beyond the position from which the next command would have been withdrawn.

Conditional functions can be achieved by use of the sign-digit selecting gates on the registers A, B, C and D.

These functions will be denoted by s(A) say, implying read out the sign-digit of the contents of A.

Thus a change to positions "n" or "m" depending upon the sign of the contents of A can be made as follows:-

$s(A) \xrightarrow{c} S$
$n \longrightarrow S$ if $(A) \geqslant 0$, shift to command in "n"
$m \longrightarrow S$ if $(A) < 0$,   "   "   "   "m"

where $\xrightarrow{c} S$ denotes the counting function of S.

Usually the third command will be replaced by commands continuing the programme, whilst the second will frequently call for a return to an earlier part of the routine.

8. Examples of Programmes

The examples to be discussed will be the same as those just considered by Professor Hartree. First, the transfer of the modulus of (4) to 6, or

$$|(4)| \longrightarrow 6.$$

This is performed by use of a single accumulator, A, and the counting and sign select functions of S and A respectively, thus:-

Command position	Command	Function
n	$(4) \longrightarrow A$	$(A) = (4)$
n + 1	$s(A) \xrightarrow{c} S$	test for $(4) \gtrless 0$
n + 2	$1 \xrightarrow{+} S$	if $(4) \geqslant 0$, shift to n + 4
n + 3	$c(A) \xrightarrow{-} A$ *	if $(4) < 0$, reverse sign of (A)
n + 4	$(A) \longrightarrow 6$	

* c(A) implies read out (A) and reset A.

This short programme may be performed in other ways but no saving in the number of commands is made.

In the second example use of more than one accumulator is made. This programme requires the summation of the contents of all positions from 100 to 140 inclusive; and the sum to be placed in 170. A is the main accumulator; D_0 is used to define the transmitting condition and C_0 is used to adjust the command which adds into A so as to traverse the correct sequence of positions:

Position	Command	Function
n	$(n + 3) \longrightarrow C$	sets adding command into C
n + 1	$40\ p_{15} \longrightarrow D_0$	sets number to determine number of cycles
n + 2	$(A) \xrightarrow{-} A$	resets A prior to starting
n + 3	$(100) \xrightarrow{+} A$	adds into A
n + 4	$1 \xrightarrow{+} C$	increases (C) by unity
n + 5	$(C) \longrightarrow n + 3$	sets (C) in place of adding command
n + 6	$1\ p_{15} \xrightarrow{-} D_0$	subtracts unit from D_0
n + 7	$s(D_0) \xrightarrow{c} S$	tests sign of (D_0)
n + 8	$-\ 6 \xrightarrow{+} S$	repeats if $\angle$ 41 additions performed
n + 9	$C(A) \longrightarrow 170$	if 41 additions complete clears (A) into 170

9. Sub-routines

It will be seen that with the aid of relative shifts from one command to another, sign discrimination in computing registers, etc., many short frequently used programmes can be put into forms which are invariant with regard to their position in the store and often also with regard to the subject data. Such programmes are called "sub-routines". These are distinct in the Mk. I computer from those programmes which cannot be put into a strictly invariant form. These latter are called "routines". A certain saving in the number of commands is made with an invariant form of programme like the "sub-routine".

10. Routines

Routines which cannot be put into a strictly invariant form are dealt with as the programme enters the computer. A special code number follows the insertion of commands which have to be adjusted and each code number defines the modification to be made. This is somewhat similar to the system of code letters used in the punching of EDSAC programmes on to tape.

11. The Incorporation of Sub-routines into Programmes

Essential to the usefulness of a sub-routine is its ease of being incorporated into an extended programme. The method adopted is to reserve the D_{15} position for storing the address to return to. This is done by two commands in the master programme which reads (S) into D_{15} and transfers control immediately to the head of the sub-routine say m. If command "n" calls for $(S) \longrightarrow D_{15}$ then n + 1 is transferred to D_{15} as a result.

The first command in the sub-routine "m" calls for the addition of 1 p_{11} to D_{15} and the last command "m + r" say transfers (D_{15}) to S, i.e. n + 1, thus restoring the sequence to the next command following the break. Thus:-

Master Programme		Sub-routine
$n - 1$,	$(S) \longrightarrow D_{15}$	$m \quad -1 \longrightarrow D_{15}$
n,	$m \longrightarrow S$	
$n + 1$,	$\boxed{\text{next command}} \longleftarrow$	$m + r, \ (D_{15}) \longrightarrow S$

This needs only four commands, two of which are in the often used sub-routine. Other methods are available.

12. Components of a Full Programme

Routines and sub-routines are stored as a library of punched cards. Usually one card holds one routine or sub-routine. At present each card is punched in binary form.

A master programme is compiled which arranges the connections to and from routines and sub-routines etc. and is punched on cards in a similar manner. The necessary library cards are extracted from file and assembled in a previously chosen order and are read into the computer. The sequence of reading data is as follows:-

1. Primary input; of 22 commands received from prewired uniselector units. This instructs the machine how to assemble data from cards.
2. Input control routine: of 16 commands, instructs machine how to deal with any controlling code punchings which follow. (c.f. EDSAC's letter code).
3. Sub-routines follow .
4. Routines inserted
5. Master programme

6. Decimal data inserted or held in reader for use during computation.

In the last case the programme calls for decimal reading and involves a decimal to binary transformation routine.

Discussion

D. R. HARTREE - drew attention to the following points:-

1. In designing any of these machines, there are two stages of operation in the arithmetical unit at which one could arrange for clearing of the accumulator to be carried out. These are immediately before putting a number into it and immediately after reading one out. The former choice means two possible instructions for each arithmetical operation, corresponding to clear and to hold, for example:-

a(i)	clear and add
a(ii)	hold and add
b(i)	clear and subtract
b(ii)	hold and subtract
c(i)	clear and count
c(ii)	hold and count, and so on;

whereas the latter choice involves only a single pair:-

d(i)	read out and clear
d(ii)	read out and hold,

and so seems more economical.

2. With regard to programming a jump by a specified number of places forward or back from the address of the current instruction, some machines have different instruct-

ions for an absolute jump (jump to a specified address) and a relative jump (jump by a specified number of places), but this is not necessary. Another method of effecting a relative jump is to add the amount (positive or negative) of the jump to the content of the sequence control register; this requires facilities for transfer of the content of this register to the arithmetical unit.

3. The meaning of the term "program parameter" appears to be interpreted differently by the C.S.I.R.O. group and the group at Cambridge. At Cambridge the "m" used in specifying the start of the set of instructions to perform the calculation such as |C (6)| to 4 is not called a program parameter but a "pre-set parameter". Its value is known before the calculation is started. However, where a value of a parameter is inserted into a sub-routine in the course of the calculation and not in reading the tape, it is known as a "program parameter" (see ref. 53).

J. C. STEWART - pointed out that in the example demonstrated on the C.S.I.R.O. Electronic Digital Computer the printing time was about 15 times as long as the calculating time and asked whether that was typical.

T. PEARCEY - said that calculating time is surprisingly long in a high speed digital computer. Printing is normally carried out as the calculations proceed. In the demonstration the printing appeared slow because the calculation was such a simple one.

Answering a further question from Mr. Stewart and a similar one from Miss Turner on what kind of checking mechanisms were used in the C.S.I.R.O. machine, he said that no provision had been made for automatic checks. Program checks as well as automatic checking mechanisms have yet to be designed. The teleprinter will eventually be used more for checking than for printing final results as this will be done on punched cards.

D. R. HARTREE - pointed out that in using a hand machine checks are made and this is equally necessary with automatic machines. Checking procedures are important and need to be programmed. However, this need for checking must not be taken to mean that automatic machines are unreliable. The EDSAC has operated for hours without electrical or mechanical breakdown.

VII. Automatic Calculating Machines and Numerical Methods

by D. R. Hartree

1. Some differences between calculations done by hand and with an automatic machine.

Although it will usually be possible to program for an automatic machine the method one would adopt in doing a calculation by hand, this method may not be the most effective to use on the machine. This arises from several differences between calculations done by hand and done with an automatic machine:-

(i) The different balance between the time required to plan a calculation and the time taken to carry out the arithmetical steps of it. In a hand calculation it is often worth spending some time working out special procedure to save arithmetic, whereas the automatic machine carries out arithmetical operations so fast that it may be more effective to use a large number of steps of a calculation involving a simple procedure rather than a small number of steps of a more complicated procedure which might be preferred for a hand calculation because it reduced the amount of arithmetic to be carried out.

(ii) The finite capacity of the store of an automatic machine, as compared with the indefinitely large capacity of the work sheets of a hand calculation. This is another reason for preferring a large number of repetitions of a simple procedure to a small number of repetitions of a more complicated procedure which would require a larger storage space for instructions. It also suggests a preference for methods which are strictly repetitive over methods which involve occasional use of special procedure for which a special set of instructions, and space in the store for them, would be required. For example, in some methods for the numerical integration of differential equations the procedure for starting the integration is different from the procedure adopted for carrying it on once it has got under way, and a special procedure is necessary if the interval of integration is changed. This is no disadvantage in a hand calculation, but all the special procedures occupy storage space in an automatic machine. In a machine with an adequate auxiliary store this will not matter; it is only serious in a machine with limited storage capacity; and even then it may not always be a serious difficulty, since although the normal practice is to read in all the instructions first and then carry out the calculations, it is possible to read in one group of instructions, carry out the calculation specified by them, and then read in another group of instructions and place them in the store in the locations previously occupied by the first group (this is called "overwriting"); any special procedure required to start a numerical integration of a differential equation could be handled in this way. But there may be other reasons in particular cases for preferring a strictly repetitive procedure.

(iii) In a hand calculation there are advantages in working with simple numerical values of some of the quantities involved, whereas with an automatic machine these advantages often do not occur. For example, in evaluating $\int_0^1 e^{x^2} dx$ it would be an advantage in a hand calculation to use simple, and equally spaced, values of x, such as $x = 0(0.1)\,1.4$, first because of the check on the values of the integrand provided by their differences, secondly because of the numerical advantage of the simple value $\delta x = 0.1$ for the interval of integration, and thirdly because of the simple values of x^2 (which could probably be written down from memory, without doing any calculation) and the fact that the values of e^{x^2} for these values of x^2 could be taken from a table without interpolating. But with an automatic machine one would probably use a sub-routine to calculate e^y from y, for example by means of the series, without using a table, and this, and multiplication by the interval of integration, is no simpler for one value of x than another; and there are other means of checking, so that there is no great advantage in the use of simple numerical values of x. Consequently methods which involve the use of values of x which have not simple numerical values, and are not necessarily equally spaced, and which are therefore unattractive for hand calculation, may be quite practicable for an automatic machine, and may be preferred if they offer other advantages. In particular for integration of a given function of x the Gauss method (ref. 53, chapter VII, §80, p.159) which offers the highest accuracy for a given number of values of the integrand, becomes attractive.

(iv) Differences in the kind of mistakes that may be made, and for which checks must be devised. Any calculation, however performed, needs checking, and the devising of adequate checks is as much a part of planning a calculation as devising means of carrying out the calculation itself; this applies equally whether the calculation is done by hand or by means of an automatic machine. Further an overall check done on final results is usually inadequate except in a calculation so short that an automatic machine would probably not be used for it; a current check is wanted, or a series of current checks, to ensure that no mistake which would vitiate subsequent work is left undetected for long. Checks may depend on the kind of mistakes which are likely to be made, and these may be different in calculations done by hand and with an automatic machine. For example, a number written into the "store" (in the form of the work sheet) of a hand calculation will not change while it is stored (though if written badly it may be misread when that number comes to be used later), whereas, with some forms of store used in automatic machines, the content of a storage location may change as a result of a defect in the storage system.

Further, the programming, as well as the arithmetic, needs to be thoroughly checked, and, as already explained (V, §3), this should be done by some means that does not involve an extravagant use of machine time.

2. Integration of Ordinary Differential Equations

I have already mentioned (I, p12), the frequent occurrence of differential equations in the quantitative treatment of a wide range of scientific and technical problems, and the consequent importance and range of application of methods of evaluating solutions of such equations.

For a first-order equation, or set of simultaneous equations, a method of the Runge-Kutta type has been worked out by S. Gill, (10). For a single equation $dy/dx = f(x,y)$ the general idea of this type of method is as follows. Suppose the integration has been carried to (x_0, y_0) and it is required to carry it through the interval n from $x = x_0$ to $x = x_1 = x_0 + h$; a number of intermediate quantities $y_a, y_b, y_c, \ldots$ are calculated, for example:-

$$y_a = y_0 + \tfrac{1}{2} h\, f(x_0, y_a)$$

$$y_b = y_0 + \tfrac{1}{4} h \left[f(x_0, y_0) + f(x_0 + \tfrac{1}{2}h, y_b) \right]$$

$$y_c = y_0 + h\, f(x_0 + \tfrac{1}{2}h, y_c)$$

$$y_d = y_0 + \tfrac{1}{2} h \left[f(x_0, y_0) + f(x_1, y_c) \right];$$

here y_c, y_d are approximations to y_1 and a better approximation can be obtained by forming a linear combination of y_c and y_d.

Methods of this kind are unattractive in many cases for hand calculation, because of all the interpolation or calculation involved in finding the several values of $f(x,y)$ required in integration through a single interval; methods based on the use of finite-difference integration formula are usually much more practicable, and easier to check. But a method of the Runge-Kutta type may be very convenient for an automatic machine. The method Gill has worked out for a set of simultaneous equations $dy_j/dx = f_j(x, y \ldots y_k)$ involves the use of only three storage locations for each equation, one for y_j, one for dy_j/dx and one for a set of intermediate quantities which can be stored successively at the same address in the course of the integration through each interval. The truncation error per interval is of order h^5., and I know no other method as accurate as this which needs as few storage locations per equation. This method has the further feature, which may in some cases be an advantage, that the calculation for each interval is independent of that for any other interval, so that there is no necessity for a number of successive intervals to be all the same length, as is necessary for most methods using finite differences.

For second-order equations with the first derivative absent, use of the formula (in central-difference notation)

$$\delta^2 y_0 = h^2 \left[y_0'' + \frac{1}{12} \delta^2 y_0'' - \frac{1}{240} \delta^4 y_0'' \right] + O(h^8)$$

provides a very convenient process, and this is one case in which a method convenient for hand calculation also forms one convenient for an automatic machine.

3. Polynomial Equations

R. A. Brooker of the Mathematical Laboratory, Cambridge, has recently made a study of various methods of using an automatic machine for the determination of the roots of polynomial equations.

$$a_0 x^n + a_1 x^{n-1} + a_2 x^{n-2} + \dots + a_{n-1} x + a_n = 0 \qquad (1)$$

and this section is a summary of the methods he studied and his conclusions.

(a) Bernoulli's Method

Let a set of numbers y_k be related by the recurrence relation

$$a_0 y_k + a_1 y_{k-1} + a_2 y_{k-2} + \dots + a_{n-1} y_{k-n+1} + a_n y_{k-n} = 0 \qquad (2)$$

the coefficients being those of equation (1). Also let

ξ_1, ξ_2, ξ_3 ... ... be the roots of equation (1), arranged in order of $|\xi|$. Then if equation (1) has no multiple roots, the general solution of the recurrence relation (2) is

$$y_k = b_1 \xi_1^k + b_2 \xi_2^k + \ldots ; \qquad (3)$$

and if $|\xi_1|$ is appreciably greater than $|\xi_2|$, then for k greater than some not too large value, the first term on the right-hand side of (3) will be dominant, and then ξ_1 is given approximately by

$$\xi_1 = y_{k+1}/y_k \quad .$$

This process fails when the equation has two dominant roots of equal modulus (for example, a pair of conjugate complex roots) but can be extended to deal with this case.

The further extension to deal with the case of a number of roots of equal, or nearly equal, modulus is complicated and involves a number of special procedures, and from the point of view of an automatic machine it is long and inconvenient.

(b) Root-squaring method.

This consists in forming successively the equations whose roots are the squares, fourth powers, eighth powers ... of those of equation (1). The coefficients in these equations become large if the root-squaring process is repeated eight or ten times, and if all the roots are required it is necessary to retain all the coefficients even though their magnitudes may vary over a range of 10^{1000}. This can only be done conveniently if the coefficients are kept in floating-binary or floating-decimal form, of which the latter is much the most convenient for output purposes and involves only a few extra instructions for arithmetical processes.

The method determines the moduli $|\xi_j|$ of the roots, but leaves considerable ambiguity in their arguments. However, if γ is a constant, which may be given any convenient value, and $z = x - \gamma$, the machine can be used to evaluate the coefficients c_n in the equation for z:-

$$c_0 z^n + c_1 z^{n-1} + \dots + c_{n-1} z + c_n = 0, \tag{4}$$

and the root-squaring process applied to this equation gives the values of $|\xi_j - \gamma|$ for the roots of equation (1). This can be done for two different values of γ, say γ_1 and γ_2; the roots are then given by the triple intersections of the circles in the x-plane with the determined values of $|\xi_j|$, $|\xi_j - \gamma_1|$ and $|\xi_j - \gamma_2|$, or by an equivalent arithmetical process.

(c) Modified Newton-Raphson method

The Newton-Raphson method for the real roots of an equation $f(x) = 0$ is an iterative method depending on the formation of the sequence (x_k) defined by

$$x_{k+1} = x_k - f(x_k)/f'(x_k). \tag{5}$$

It can also be used to find the complex roots, provided that the process is started from a complex value x_0 (this proviso is unnecessary if the coefficients in equation (1) are complex). Brooker modifies this formula by writing

$$x_{k+1} = x_k - p\, f(x_n) / f'(x_k) \qquad (6)$$

and making p adjustable in the course of the calculation according to certain criteria; if these indicate that the value of x_{k+1} given by formula (5) has overshot the root, then a value of p less than 1 is taken in (6); if the criteria indicate a slow convergence with $p = 1$, a greater value is taken. It is of interest that (6) is a second-order formula for a root of multiplicity p.

(d) Minimisation methods

We can attempt to find a solution of the equation $f(z) = 0$ (where $z = x + iy$) by starting from some arbitrarily chosen value of z and making successive changes in x and y alternately in such a way as always to decrease $|f(z)|^2$; it can be shown that if f(z) is analytic the only minima of $|f(z)|^2$ are its zeros, and these are the required roots. This can be applied in particular to polynomial equations.

Another way of finding the minima of $|f(z)|^2$ is to follow out approximately a curve of "steepest descent" of this function, that is to say, to make successive increments of z each in the local direction of maximum decrease of $|f(z)|^2$.

Brooker has tried all these methods on the EDSAC and come to the conclusion that the modified Newton-Raphson process is the best as a practical general method for an automatic machine.

4. "Monte Carlo" processes

The term "Monte Carlo" process has been used to denote a method by which a statistical sampling process is used to obtain an approximate result for a numerical problem which is not in itself statistical in character.

For example, suppose $f(x)$ is a function such that $0 \leqslant f(x) \leqslant 1$ for $0 \leqslant x \leqslant 1$, and it is required to evaluate $\int_0^1 f(x)dx$ (see fig. 1). Let (x,y) be a pair of random numbers; then if $y - f(x)$ is positive, the point (x,y) does not lie in the area representing the integral to be evaluated, whereas if $y - f(x)$ is negative, the point (x,y) does lie in this area. If we take a large number of pairs (x,y), randomly distributed, then

$$\left.\begin{array}{c}\text{Area representing}\\ \int f(x)dx\end{array}\right\} = \frac{\text{Number of pairs for which } [y-f(x)] \text{ is negative}}{\text{Total number of pairs}}$$

approximately.

This is not a method which would be adopted for evaluation of a single integral, but might be very effective for the evaluation of multiple integrals; it has been estimated that it might be the best method for n-fold integrals with n greater than about 5.

Another example is provided by various applications of a "random walk" process. One such process is as follows.

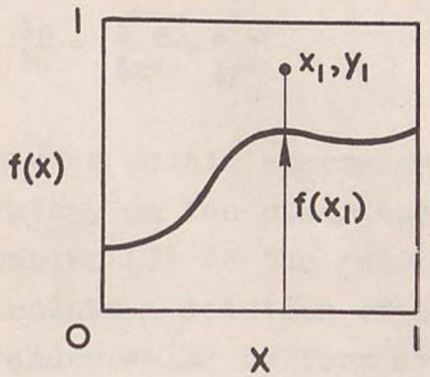


FIG. 1. EVALUATION OF $\int_0^1 f(x)\,dx$ BY THE "MONTE CARLO" PROCESS.

A point moves by successive steps of length a in the (x,y) plane, steps from (x,y) to $(x \pm a, y)$ and to $(x, y \pm a)$ being equally probable, uncorrelated with the direction of the previous step, and being taken at equal intervals of time t. The distribution of the probability of the moving point being at different positions of the (x,y) plane after n steps satisfies a finite-difference form of the heat conduction equation:-

$$\frac{\partial \theta}{\partial t} = \frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2} \qquad (7)$$

This can be used to obtain approximate solutions of the random walk problem. On the other hand, recognising the relation of equation (7) to the random walk problem, we might try to calculate a solution of equation (7) by evaluating enough random walks to form an adequate sample, say 10,000 walks of 1000 steps each.

These are simple examples, but they illustrate two points:-

(i) To obtain an adequate sample, a large number of cases have to be evaluated, and one would hardly contemplate doing a calculation on such a scale without the facilities which an automatic digital machine provides for doing it.

(ii) In order to be applied to such a calculation, the machine must be provided with either a table of random numbers or means of generating them.

Discussion

T. M. CHERRY - referred to the importance of calculations in the complex domain and said that he hoped designers were giving adequate attention to this matter. He was conscious of the fact that complex numbers could be represented and manipulated in standard machines by appropriate sub-routines. Professor Cherry then commented on the matter of checking and pointed out that checking by programming the problem for solution by a different method was standard practice in most computational work.

J. C. JAEGER - said that his main interest at this stage was with the effect of the high-speed automatic machines on computational methods generally and he was also anxious to hear some expression of opinion on the future scope of hand calculating machines. With regard to his first point he added that in his own view it appeared that the new machines would encourage more study and research on iterative methods in numerical mathematics.

D. R. HARTREE - in replying to Professor Jaeger's second question, said that as yet it was too early to forecast accurately the eventual role of the various types of calculating machines. However, he expressed the opinion that the advent of new, fast machines would open up so many new fields that existing machines would be required more than ever. Most exploratory work will still be done by hand machines. Punched card machines have special fields of usefulness in particular problems requiring sorting.

D. M. MYERS - in continuing the discussion, pointed out that when mathematical equations are broken down into a form suitable for a digital computer there will be a requirement for the computer to deal with exponentials, reciprocals and so on which involve division. Multiplication with binary digits can be achieved by repetitive addition together with instructions to shift and to examine individual digits of the multiplier. It can be shown that division in the binary form can be carried out by a somewhat similar sequence of instructions and comparatively little extra equipment would be required to do this automatically. In spite of this, in the EDSAC for example, designers and others leave division to be programmed. Professor Myers asked Professor Hartree if he would comment on this and also explain briefly the method of programming a division.

D. R. HARTREE - said that the question of putting in a division unit was an engineering one, and, apart from the purpose of simplifying the engineering, he did not recall why one had not been incorporated in the EDSAC.

Professor Hartree said that there were a number of methods of programming a division and certain divisions were better programmed by one than another. He could not recall them all in detail but mentioned a recurrence relation used for calculating a reciprocal. The reciprocal of a number "a" may be obtained by successive evaluations of

$$y_k = y_{k-1}\,(2 - ay_{k-1})$$

For calculation of exponentials use could be made of the relationship

$$e^{a+b+c} = e^a e^b e^c$$

A more generally used method is illustrated with the circular functions. For $|x| < \pi/2$, 10 decimal digit accuracy is given by the first 17 terms of the Taylor's series for sin x. If only a limited range of x is required, an expression of equal accuracy with fewer terms is given by an expansion in Tchebyscheff polynomials

$$T_n(x) = \cos (n \cos^{-1} x)$$

T. PEARCEY - added a point to the discussion on the employment of division units when he pointed out that complications arise in the determination of the binary point when divisions are being carried out.

R. T. REYNOLDS - said that certain I.B.M. machines carried out division automatically by the method suggested by Professor Myers.

Mr. Reynolds took this opportunity to point out a misunderstanding in the use of the word Hollerith in relation to punched card equipment. It is in fact a trade name used for one make of punch card equipment and not to be confused with equipment such as that produced by I.B.M.

VIII. Programming for Punched Card Machines

by T. Pearcey

Before discussing punched card methods, I would like to bring to your attention certain of the multi-register equipment kindly exhibited by some commercial organisations. Most of the devices contain a number of registers easily capable of accumulating information transferred to them. Usually about six registers are available. In most cases a single extra register, called a "cross-footer" is capable of subtraction as well as addition, and usually a number can be transferred and added to the cross-foot register from the keyboard or from any other register, and thence transferred, added or subtracted, to any other register. During this process, printing of the transferred data can be made and the transmitting register may either reset or not as desired. A sequence of such operations can be set up and is put into the machine as a programme of action by suitable shaped "stops" placed along a "control bar". The operation can usually be made automatic, as will be seen in the "Sunstrand Model D" shown.

The "National Type 3000" is a six register machine, which is capable of transmitting out of any one register and receiving and adding into any group of the other five registers. In particular two of these registers can subtract and thereby act as cross-footer registers. A programme of operations is provided in a non-automatic fashion. The reception of data is determined by the arrangement of "stops" upon a "control bar", whilst the transmission and transmitting register is specified by the operation of depressing certain keys at each stage of the programme. Whilst not fully automatic the number of programme steps required to arrive at the same arithmetical result on this type of machine is considerably less than on any other multi-register machine. The two National machines exhibited are set up to perform the acts of differencing and sub-tabulation.

The "Burroughs M 200" model exhibited is a six register device with a single cross-foot register. It also possesses the facility for multiplication, and has the advantage that all functions can, for test purposes, be made via the keyboard with a skeleton "control bar". The inclusion of a multiplying device improves the flexibility of the machine. The model exhibited has been set up to tabulate the solution of a second order differential equation of the normal form:

$$Y'' + P(x)Y = 0$$

using the difference expression

$$Y_0 = h^2 \left[\delta^{-2} Y''_0 + \frac{1}{12} Y''_0 - \frac{1}{240} \delta^2 Y''_0 \right]$$

in which $\delta^2 Y''_0$ is extrapolated to get a Y''_0 and corrected on completing the cycle of computation. No auxiliary machine such as a Brunsviga is used.

The availability of a multiplying device extends the effective use of a multi-register machine greatly, and experience seems to show that whilst six registers is the best capacity for machines without multiplication, the inclusion of a multiplier raises the capacity required to about ten registers.

One particular model of punched card machines must be mentioned; that is the newly arrived I.B.M. type 602 A calculating punch. This device has been designed largely for scientific work. It contains 80 counting wheels and eight 12-digit storage registers and is fitted with fully automatic multiplication and division functions. The next important feature of the device however is its programming properties, which follow the principles of programming, dealt with this morning. Following the feeding of a card and the consequent reading of data into registers and counters the machine passes through a sequence of separate stages known as "Programme steps". During each programme step calculating functions are performed in a manner controlled by the setup of an electrical connection plugboard. Twelve such steps are provided for in the machine; but the device is so flexible that, following a card feed action, any one of the twelve steps may be called up, after which the functions are traversed in sequence until a further card feed action is called for. Programme steps may be skipped and recalled at will, and the number of steps may in effect be increased far beyond the twelve steps provided by calling electrical relays into action whose contacts are plugged to have the effect of rearranging the plugging, and thus the function to be called in the following steps. Thus, at step 12, relays may be called up and step 1 may be called to follow, and as the steps 1 to 12 are traversed a second time, fresh functions are provided by the new state of the relays called upon on step 12, and held for the following step. This device is important in the development of low speed automatic computer, and is immensely powerful for purposes of automatic computation.

The example which has been set up on the 602 A machine exhibits the property of recalling programme steps, according to a condition of the results of current calculations. It will be seen computing square roots by a well known iterative process.

$$X_{n+1} = \tfrac{1}{2}(X_n + N/X_n); \quad \underset{n=0}{\mathrm{Lt}} \; (X_n^2 - N) = 0$$

A number N is fed via a card, and N/2 is used as the first approximation, X_0, to the square root. By a division and addition and a multiplication by $\frac{1}{2}$ the next approximation X_{n+1} is found from an approximation X_n whence $X_{n+1} - X_n$ is calculated. This is negative unless $X_{n+1} = X_n$ whence the sequence may be stopped. This discrimination on the sign of a result decides whether or not the result is correct and is to be punched; the counters then being reset to zero, the next card fed.

To proceed now to punched cards in particular. I shall try to indicate to how much detail a computing problem must be considered in its relation to the available operations provided by punched card machines and by describing in detail how to do what appears to be a very simple computing job. The sequence of operations may appear somewhat complicated but is not so in practice once the operator gets familiar with the routine.

I must first discuss the particular features of punched card methods and the functions of the machines themselves, and my remarks will apply in particular to the machines used in the Division of Radiophysics of the C.S.I.R.O.

A punched card computing system consists of a group of separate and distinct machines. Each machine is designed to perform a group of functions and the actual actions which it performs are controlled by the network set up on an electrical plugboard. The plugboards are removable and by replacing one by another the functions of any one machine can quickly be readjusted to suit different needs. The standard machines were originally designed for commercial accounting purposes, but have recently been taken over and adapted to scientific computing.

A card about 8" x 3" in dimensions is punched with suitably positioned holes to represent decimally coded digits, and up to eighty such digits can be held on one card. A single card therefore acts as a simple store for information which may be transferred from one place to another. More than one number can be punched into any one card, but it is usual to place closely related data on the same card.

Consider for example a file of cards used to represent a tabulated function. Thus suppose a table of the function f(x) of the independent variable x be recorded at intervals $x = 0$ to $x = 1$ say at intervals of 10^{-3}. We should punch the numbers 0, 1, 2, ... to 1000 corresponding to $x.10^3$ on each of 1001 separate cards in the same columns, say 1, 2, 3 and 4. Then on the top card we punch the value of f(0), the next f(0.001), f_1 say, and so on to f_{1000} at $x = 1$ on the same columns from 4 onwards, say. We could also punch the even central differences say $\delta_n^{\,2}, \delta_n^{\,4}$ on the card designated by n in the first four columns, and the file could then be used for purposes of interpolation to values of x lying between the tabular entries. Such a file of cards is a store of a function table and is called a "master file".

To come back to the machines; the function of each machine is to accept a card from a file provided to it and to perform a limited but controlled sequence of acts upon the data held on the card accepted, and then to feed another card and repeat the performance and so on. The action of each machine might be likened to the traverse of a sub-routine of a larger programme over and over again. Thus the machine performs the specific functions, often repeated, of the nature of sub-routine functions whilst the human operator who removes or replugs plugboards and passes files of cards from one machine to another has the effect of connecting sub-routines together in a prescribed fashion particular to the full computation, and thereby take the place of the master programme in an automatic computer.

There is a striking difference between the use of groups of punched card machines and an automatic computer, and that is due to the fact that a card once passed through a machine cannot be recalled, and that it is economical to pass many cards through a card machine for each plugboard setup. Further the amount of effective storage on a card file is unlimited.

Thus, because in an automatic computer the storage capacity is limited by the size of the machine, a lengthy computation would be made by passing immediately from one sub-routine to another until the final result is obtained and then recommencing with fresh data. This corresponds to working with pencil and paper and desk machines in a "horizontal" manner over the worksheet. With punched card machines however, a large block of data stored in a sufficiently large card file is dealt with item by item in a strictly repetitive manner; this corresponds to working "vertically" down a work sheet instead of "horizontally".

Any computation must be moulded into a form for punched card work, so that "vertical" working is possible. As an example of how this affects the approach to a problem, consider the solution of a set of differential equations of the form

$$\frac{dy}{dx} + \varphi(y, \lambda) = \psi(x, \mu)$$

where φ and ψ functions not necessarily linear in y or x, and where λ and μ are parameters, for a range of which solutions are required. Normally we should treat each parameter pair separately, and solve each equation by step by step processes. Suppose we require 50 values of λ and 100 values of μ that is, 5000 solutions, a fairly large task.

We should treat these by punched card methods, all at the same time. We should construct tables of the functions φ and ψ, for the various values of λ and μ over the expected range of y and x, on cards in a manner described above. These could frequently be computed by punched card machines themselves. Then we could select from these tables the appropriate value of φ and ψ for the various values of λ and μ in pairs at the commencement of a step of integration, calculate $\psi - \varphi$ and multiply by Δx, the interval of x to obtain increment in y_{jx}, $\Delta y^{(n)}_{jx}$ for values λ_k, μ_j

$$\Delta y^{(n)}_{jk} = \Delta x^{(n)} \left[\psi(x^n \mu_j) - \varphi(y^{(n)}_{jk} \lambda_k)\right]$$

and

$$y^{(n+1)}_{jk} = y^{(n)}_{jk} + \Delta y^{(n)}_{jk}$$

All 5000 equations could be treated for a single step of integration at the same time. Repetition of the process would continue the integration.

Turn now to the actual functions performed by the punched card machine.

2. The Sorter

Sorting is an essential feature of punched card manipulation, since the data on one card is related to the data on one or more other cards, and the data must be arranged in a prescribed order so that a regular sequence of action made on the contents of the top card can be directly related to similar actions made on the contents of the next card and so on. The sorter has a device which accepts a card file, card by card inspects the digit punched on a prescribed column and passes each card into one of 12 pockets according as the punched hole corresponds to a digit 0 - 9 or X or Y. These last two punch positions lie above the 0 position. By repeated sorting and handling a file of cards may be ordered in ascending or descending sequence of the data in any one field of columns.

3. The Reproducer

The reproducer is a device whereby selected data from one card file may be transferred card by card on to a second card file. It is provided with two independent card feeding hoppers - one for reading and one for punching. See fig. 1. Cards are fed in similar positions from both feed stations and pass under control reading brushes which detect the presence of X punches. Such punches may be used to control functions at later times. Cards A,A' pass on to B,B'; that on the reading side passing through an 80 column reading station and that on the punching side through an 80 column punching station where punching takes place. The cards then proceed to C,C' each through a reading

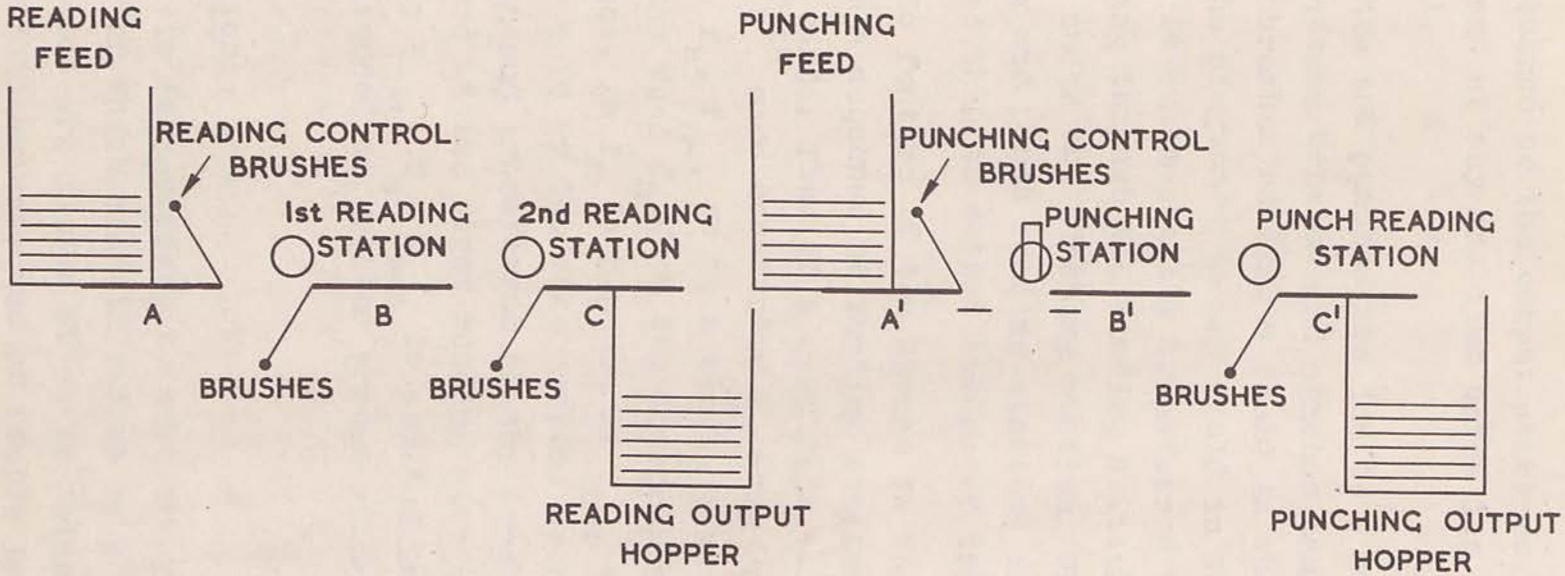


FIG. I. THE REPRODUCER.

station, and thence to the output stackers. Cards are fed continuously and at any one time cards lie in all positions ABC and A'B'C'.

All brushes and punches are led to a plugboard where interwiring between brushes and punches can be made. Thus by wiring the brushes reading a field in the final reading station via the plugboard to any field in the punching station, data is automatically transferred from the field of the card passing through the reading station and is punched onto the card under the punching station. Normally the second reading and punch reading stations are wired to read and compare and thus to detect errors of transfer.

A valuable feature of the device is that data may be assembled from a sequence of reading station cards onto punch station cards. Thus if a card file is sorted into sequence, the n^{th} card containing f_n say, and we wish to assemble f_{n+1}, f_n, f_{n-1} on to a single card for each value of n then we may read f_n from the second reading brushes onto the punches. As f_n is punched so f_{n-1} which is punched onto the prior card by the same action, is read at the punch reading station and transferred to the punches. Further f_{n+1} may be read at the first reading station and punched, thus eventually f_{n+1}, f_n, f_{n-1} is punched onto each card. This may be extended to greater groups of data.

4. The Collator.

Sometimes it is necessary to sort two files of cards together, both of which contain values of a variable "x" say. If the fields are chosen so as to coincide the sorting is sufficient. This coincidence of fields is however not always possible or convenient, in which case the collator is necessary. The collator provides primarily the function of merging two card files. This is done by reading a designation from one field on the top card of the first or primary file, and the corresponding field from the top card of the second or secondary file possibly from a different set of columns. The values are then compared one against the other. The collator may be plugged so as to feed primary cards whilst their values are less than and possibly equal to the next secondary value, and to feed secondaries whilst their values are lower than the next primary and so on. This forms a uniformly ascending sequence out of the two separate packs. Each field must be previously sorted in order of the variable.

5. The Tabulator

The tabulator is the main computing device. In our case it possesses six counters. Each counter can accumulate from a card, as a card is read via a pair of successively placed 80 column reading stations. Following one or more card feed operations a sequence of internal functions can be performed usually a succession of up to eight steps. During each step a set of additions, and/or subtractions between selected counters can take place, according to a plugboard set-up.

In our case certain modifications have been made by which the steps may extend beyond eight, may be recalled or repeated either indefinitely, or according to the sign of the content of any counter. Counters can be made to add into themselves. This may be done on standard machines, although to simplify plugboard wiring the function of duplications' has been added.

Counters may Transmit out (denoted by T)
Subtract out (denoted by S, i.e., transmit a complement)

Accept (denoted by A, i.e., add in)
Reset (denoted by C)

Results from any or all counters may be printed out onto a sheet of paper and may be punched out eventually onto a card. The counters are connected to the punches of a reproducer in order to perform the operation known as "summary punching".

6. Subtabulation to halves

As an example of the combinations of all these devices consider the problem of subtabulation of a function table to halves. Suppose we are provided with a long list of values of a function f(x) at values x or at uniform intervals say h, and that we wish to form a table containing all values of f(x+h/2). We assume that the table is such that 4th differences are less than 10^3 units in the last decimal so that throw-back methods are valid.

The method to be adopted will be that using differences. The advantage is that errors are detected easily wherever made. The expression used is

$$f_{\frac{1}{2}} = \tfrac{1}{2}\left[(f_1 + f_0) - \tfrac{1}{8}(M_1^2 + M_0^2)\right]$$

where

$$M_0^2 = \delta_0^2 - 0{\cdot}184\,\delta_0^4$$
$$M_1^2 = \delta_1^2 - 0{\cdot}184\,\delta_1^4$$

are the modified second differences of f.

We have first to compute the differences, then convert the δ^4 to $0.184\delta^4$ and hence obtain M^2 from δ^2. Then the data to compute $f_{1/2}$ must be assembled and $f_{1/2}$ found.

We first punch the successive values of f and the associated value of x onto cards, one card per entry, using the same fields on each card. The x's and f's should then be printed out on the tabulator and proof read and corrected if necessary. Any errors missed will be detected and corrected later.

Next the file, now called file A, is sorted into sequence on the field "x". These are fed through the tabulator in reverse order and the differences of successive values computed. The even differences will be independent of the backward ordering of the file and are of the same sign as if the file were passed from the front. Fig. 2 shows the inter counter transfers needed to compute differences on the tabulator.

Since only the backward row of differences can be calculated on the data from the last card passed, the differences obtained at each stage will be the advancing diagonal differences of the table f as if it were tabulated from the front. Second and fourth differences are summary punched together with f and the value of x, at the same time the f value from the previous punched card passing the punch reading brushes is punched onto the following card on the reproducer. We thus obtain on a summary card corresponding to a function f_0 and variable x_0 the values:

$$x_0, f_0; \Delta_0^2, \Delta_0^4; f_1$$

$$\text{or} \quad x_0, f_0; \delta_1^2, \delta_2^4; f_1$$

the value f_1 being obtained from the previous card punched.

This newly punched file will be called the B file.

The δ^4's must now be associated with $0.184\delta^4$. Since $\delta^4 < 1000$, a master file in the form of an initial table of $0.184\delta^4$ may be compiled, a file say C, of 184 cards only, with the following sequence of data:

designation $0.184\delta^4$	critical δ^4
0	0
1	3
2	9
3	14
etc.	etc.

together with an X punch on say column 80, so that these cards may be distinguished from cards of file B as constructed. Such a file is of frequent use and would normally exist in the files of a computing laboratory. File C is sorted into sequence of field δ^4.

File B is now sorted into sequence of field δ^4 and is passed through the collator as the secondary file, whilst file C is used as the primary file, and both are collated or merged in the field δ^4. The first card of file C heads the combined file with the B cards for which $\delta^4 = 0, 1, 2$, following. Then follows the next card of C followed by B with $\delta^4 = 3, 4, 5, 6, 7, 8$ and so on.

The combined file is now passed through the reproducer punch feed. The X punched on column 80 on the C cards are detected by the control brushes, which are plugged so as to stop the punching on X punched cards, i.e., C cards. As a C card passes through the punch reading brushes, the value of

$.184\delta^4$ is transferred to the following B card. As this B card proceeds past the reading station the $.184\delta^4$ is punched onto the following B card on the same field. This goes on until a O card is detected when the punching on this card is inhibited and the new value of $.184\delta^4$ is read off this O card onto the following B card and so on. On our B cards we now have the data

$$x_0, f_0, \delta_1^{\ 2}, \delta_2^{\ 4}, f_1, 0.184\delta_2^{\ 4}$$

where $.184\delta_2^{\ 4}$ is correct to the nearest unit.

The O file is sorted out on column 80 and the B file resorted into sequence in x, and a reproducer run assembles the information ready for the computation of $f_{\frac{1}{2}}$.

For this we arrange to read δ^2 from the previous card and punch it onto the next card via the punch reading brushes, and to transfer $.184\delta^4$ twice by this means. Thus $0.184\delta_0^{\ 4}$ is transferred from the x_0 card to the x_1 card and thence to the x_2 card. We shall now have the following data on the x_0 card:

$$x_0, f_0, \delta_1^{\ 2}, \delta_2^{\ 4}, f_1, 0.184\delta_2^{\ 4}, \delta_0^{\ 2}, 0.184\delta_1^{\ 4}, 0.184\delta_0^{\ 4}$$

of which f_0, f_1; $\delta_0^{\ 2}$, $\delta_1^{\ 2}$, $0.184\delta_0^{\ 4}$ and $0.184\delta_1^{\ 4}$ are to be used.

A tabulator run now computes $f_{n+\frac{1}{2}}$ which may be summary punched onto a file D together with its associated $x_n + h/2$ value which may be calculated at the same time. Both x and f are punched in the same fields as are the corresponding values in file A. Fig. 3 shows the transfers involved, the duplication function being used to obtain the product $0.0625\ (M_1^{\,2} + M_0^{\,2})$ and $0.8\ (f_1 + f_2)$

The file D is merged with the file A in the field x by sorting and the combined file is differenced by the tabulator and the differences printed for even checking.

Discussion

T. PEARCEY - in reply to a question concerning the types of problem which the C.S.I.R.O. punched card machine had been used to solve, said that it had been used for summations of Fourier series, numerical differentiation and integration as well as for the construction of function tables.

D. R. HARTREE - Mr. Pearcey described the "vertical" manner in which operations are carried out with punched card machines. It appears that herein lies their strength and also their weakness. They are useful if one operation on a whole set of data can be carried out without reference to previous results, otherwise they are not so suitable.

It was mentioned that a round-off should be added for positive numbers and subtracted for negative numbers. He asked Mr. Pearcey whether the round-off should not be added for negative numbers also.

T. PEARCEY - replied that the reason for the procedure given is that the machine is made to deal with negative numbers represented as 9's complements, not 10's complements.

D. R. HARTREE - Mr. Pearcey mentioned an actuarial reference for the modification of differences by the "throw back" method. Another reference which would probably be more accessible here is the second volume of Chambers' 6-Figure Mathematical Tables (1950).

Professor Hartree then made a general remark concerning the complexity called for and ingenuity required to program relatively simple problems on punched card machines. He felt that such problems could be done much more simply on an electronic machine and asked representatives of the makers of punched card machines what they thought.

T. R. REYNOLDS - agreed with Professor Hartree and said that his company realised this, for they are developing electronic machines. The Sequence Controlled Electronic Calculator is already in production.

T. PEARCEY - replying to a question on why the sorter is not used for merging, said that functions to be used for merging may be on different packs. Use of the reproducer to put them on the one field would be too wasteful in time.

In reply to another question on the desirability of using optimum interval tables, Mr. Pearcey said that because of the general value of master tables for other purposes, special tables with their extra figures would be uneconomical.

Concluding the discussion, Mr. Pearcey said it was clear that the advent of the high speed automatic machines had restricted the role of punched card machines. Nevertheless, there were many jobs for which they were suitable, especially in view of the fact that there will be a long list of complex problems waiting for the electronic machines.

C O U N T E R S :

		1	2	3	4	5	6
Initial Content		f_1	$-\Delta_1$	1	$-\Delta_1^3$	0	0
Card Feed							enter f_o and x_o
Step	Operation				SC $\longrightarrow$	A	
1	Content				0	$-\Delta_1^3$	
Step	Operation			SC $\longrightarrow$	A $\longrightarrow$	A	
2	Content			0	$-\Delta_1^2$	$-(\Delta_1^2-\Delta_1^3)$	
Step	Operation		SC $\longrightarrow$	A $\longrightarrow$	A $\longrightarrow$	A	
3	Content		0	$+\Delta_1$	$-(-\Delta_1+\Delta_1^2)$	$-(-\Delta_1+\Delta_1^2-\Delta_1^3)$	
Step	Operation	SC $\longrightarrow$	A $\longrightarrow$	A $\longrightarrow$	A $\longrightarrow$	A	
4	Content	0	$-f_o$	$-(f_1-\Delta_1)$	$-(f_1-\Delta_1+\Delta_1^2)$	$-(f_1-\Delta_1+\Delta_1^2-\Delta_1^3)$	
Step	Operation	A $\longleftarrow$	A $\longleftarrow$	A $\longleftarrow$	A $\longleftarrow$	A $\longleftarrow$	T f_o only
5	Content	f_o	$-\Delta_o$	Δ_o^2	$-\Delta_o^3$	Δ_o^4	f_o, x_o
Step	Operation	Print f_o	Print Δ_o	Print Δ_o^2	Print Δ_o^3	Print Δ_o^4	Print x_o, f_o
6	Content	f_o	$-\Delta_o$	Δ_o^2	$-\Delta_o^3$	Δ_o^4	f_o, x_o

		1	2	3	4	5	6
Step	Operation			Punch		Punch	Punch
7	Content	f_o	$-\Delta_o$	$\Delta_o^2 (=\delta_o^2)$	$-\Delta_o^3$	$\Delta_o^4 (=\delta_2^4)$	x_o, f_o
Step	Operation					C	C
8	Content	f_o	$-\Delta_o$	Δ_o^2	$-\Delta_o^3$	0	0
Return to							
card feed							

Fig. 2 Computation of Differences by Tabulator

Table f(x) is entered in descending sequence of x.

Expressions used are:

$$\Delta_o^4 = f_o - (f_1 - \Delta_1 + \Delta_1^2 - \Delta_1^3)$$

$$-\Delta_o^3 = f_o - (f_1 - \Delta_1 + \Delta_1^2)$$

$$\Delta_o^2 = f_o - (f_1 - \Delta_1)$$

$$-\Delta_o = f_o - f_1$$

COUNTERS

	1	2	3	4	5	6
Initial Content	0	0	0	0	0	0
Card	Enter	Enter	Enter	Enter	Enter	Enter
Feed	1000 f_1	1000 f_o	δ_o^2	δ_1^2	$0.184\delta_o^4$	$0.184\delta_1^4$
Step	A ←	T	A ←	A ←	SC	SC
1	$1000(f_1+f_o)$		M^2	M^2	0	0
Step		C	A ←	T		
2		0	$(M_o^2+M_1^2)$			
Step	D.T. →	A	D.T. →	A		
3	$1000(f_1+f_o)$	$1000(f_1+f_o)$	$2(M_o^2+M_1^2)$	$(M_o^2+M_1^2)$		
Step	D	A	D.T. → x10	x100 → A	→ A	
4	$4000(f_1+f_o)$	Add 500 to round off	$4(M_o^2+M_1^2)$	$21(M_o^2+M_1^2)$	$200(M_o^2+M_1^2)$	
Step	A ←	T	T → x100	→ A	A	
5	$5000(f_1+f_o)$			$25(M_o^2+M_1^2)$	$600(M_o^2+M_1^2)$	

COUNTERS

	1	2	3	4	5	6
Step	A ←	←	←	SC		
6	$5000(f_1+f_o)$ $-25(M_o^2+M_1^2)$			0		
Step	A ←	←	←	←	SC	
7	$1000f_{\frac{1}{2}}x500$				0	
Step 8	Punch $1000f_{\frac{1}{2}}x500$ omitting three lowest digits					
Step	C	C	C	C	C	C
9	0	0	0	0	0	0
Return to card feed						

Fig. 3 Computation of $f_{\frac{1}{2}}$ by Tabulator

$$f_{\frac{1}{2}} = \frac{1}{2}\left[f_1 + f_o - \frac{1}{8} \left(\delta_o^{\,2} + \delta_1^{\,2} - 0.184\delta_o^{\,4} + \delta_1^{\,4}\right)\right]$$

Multiplication by 10 and 100 during transfers achieved by
a sideways shift of digits into receiving counters
Duplication is indicated by "D"

IX. The Functional Design of an Automatic Computer

by T. Pearcey

Before proceeding to any discussion of the pros and cons of various particular methods of design for automatic computers let us first study what is the general organization of a computer, which shall possess all the necessary and simplest properties, but only such as are sufficient to supply the needs. Upon the basis of such a discussion we may build up a design for a computer of more practical utility.

First, what is a computer to consist of and what actions must it in general perform? We have already in the past two days seen by actual examples, what is to be expected of a computer, one salient point being that for commands to be amenable to arithmetical adjustment during the course of a computation, both commands and subject matter must be held in a store of the same kind, and that by consequence commands are recorded in essentially the same manner as is the subject data. Both commands and subject data are subject to different conventions implicit in the design of the computer.

A computer must contain a main store consisting of a group of registers which will contain both commands and subject data. Each position will be associated with a serial number and will be capable of reading out its digit contents and also of substituting fresh data according to any current command. The operation of the computer will consist of a sequence of organised transfers to and fro of the contents of selected store positions between the store and a small group of registers outside the store. In order that a computer may proceed data must be associated or transformed during transit from one register to another.

The sequence of transfers will be determined by a special component of the computer which provides the overall controlling functions. The computer will thereby perform a repeated standard set of operations which will not vary. During the performance of each set of operations a command must be selected from store, suitably treated to determine its implications and its demands satisfied, when a further command may be selected and so on. Only one command will be accepted and satisfied during any one basic set of operations. This set of operations we may call the "computer routine".

The main component of the computer will consist therefore of the "store" to which is directly connected some device for selecting any one of the store registers for transfer, that is a "store selector". A single register may be included called the "function register", with which are associated transfer gates which define the transformations and transfers which take place between store and "function register"; these are the "function gates". A register must be provided for holding the current command; this is the "interpreter register". To this will be attached a "function selector" which shall decode part of the command and thereby select the appropriate function gate for transfer. A further register will be necessary in order that a position selected in the store may be held in action over the period of transfer. This is the "store control register". Certain gates are necessary for transferring data into the store control register.

Two further registers and their associated gates will be needed. First the "input register", whereby data may be entered into the computer and an "output register" whereby data may be transmitted out.

We shall assume that the number of available store positions is so large that we need not consider need for economy in storage space, and that the binary scale is adopted, thus reducing the complexity of the necessary functions to a minimum.

2. Computing Functions

We shall presume that the computer is provided only with the minimum of the simplest possible functions. This implies that neither addition, multiplication nor counting is a permissible function in this theoretical design since all functions can be constructed from only two more fundamental logical functions.

These logical functions are those of conjunction and disjunction. The first of these is likened to digit by digit multiplication. Disjunction is otherwise known as logical summation. If propositions A and B possess truth values either 0 or 1, then their conjunction (C) and disjunction (D) satisfy the following four relations:

A	B	C	D
0	0	0	0
0	1	0	1
1	0	0	1
1	1	1	1

From these relationships, any arithmetic, and in particular binary arithmetic may be constructed. I shall not enter here into the details of such constructions but it must be possible to construct a programme, using these and one or two other functions so that arithmetic may be performed.

A special function which is essential to the fully automatic function of the machine is a discriminative test of some kind whereby decisions may be made concerning the course of the computation through the programme according to the results of past computations. In the case of this machine, since there is no direct arithmetical function a test must be made of the presence or otherwise of any digits held in the function register. According as one or more digits are held or none are held so the selection of the next command may be made.

3. Address Code

The single address code system, in which commands are accepted serially, possesses simplicity from an engineering point of view but implies the presence of counting and adding functions in the machine. Since such functions do not apply to the theoretical device, we are forced to adopt a two address system. A command will be thought of as consisting of three parts corresponding to numbers f, m and n. The f-number will define the function to be performed; the m-number will define the store address to which the f-number refers, except in the conditional case; whilst the n-number will define the address of the next command to be accepted, except again in the conditional case.

If the f-number calls for the conditional function, an affirmative condition will mean that the n-number is to retain its usual meaning; whilst if in the negative, the m-number is to take the place of the n-number; then "n" denotes the location of the next command.

4. Detailed Functions

Not more than seven separate functions are required.

They are together with a symbolic representation, as follows:

1. Dm; replace the content of the function register by its disjunction with regard to the content of the store location "m".
2. Cm; replace the content of the function register by its conjunction with regard to the content of the store location "m".
3. Rm; transfer the content of the function register to store location "m" and clear the function register.
4. Im; transfer the content of input register to store location "m" and reset the input register, and set up next datum.
5. Om; reset the output register and transfer the content of the store location "m" to the output register and record.
6. Zm; test the content of the function register for presence of non-zero digits. If non-zero accept the next command from store location "m".
7. T; a command signifying "cease operation".

The last of these commands is not essential, but may be considered necessary to the fully automatic functioning of the machine.

5. Organisation of the Computer

Fig. 1 shows a block schematic diagram of the computer possessing the functions listed. Selector units are drawn as triangular figures. Function signal outputs numbered 0 to 7 are taken to the similarly numbered inputs in the function gates D, C, R, I, O, Z, and T; (number 6 being unused).

The control unit provides signals a to h, which are of standard wave form shapes and are taken to controlling gates marked with the same letters.

The sequence of functions is now as follows; commencing from the point following the completion of the operation satisfying the last command.

i. Normally the signal "a" occurs first and transfers the number "n" or "m" through the gates A or B respectively to the store control register. Suppose "n" is so transferred. Then the store selector decodes the number "n" and prepares position numbered "n" in the store for action.

ii. The "n" address is held in the store control register and the interpreter register is reset by the signal "b".

iii. The gate "E" is opened by signal "c", so that the contents of store position "n" is transferred to the interpreter register.

iv. The store control register is now reset by signal "d".

v. The "m" address is now transferred via gate B from the units position register to the store control register by means of signal "e".

vi. The signal "f" is now applied to the function selector outputs to activate the selected function gate so that a transfer from or to store may be made, or in the cases Z and T, for special actions to take place.

vii. The store control register is again reset by signal d, and the sequence returns to stage 1 to repeat the cycle of operations.

This cycle is modified only if T or Z functions are called. In the case of a T function a signal "g" is allowed

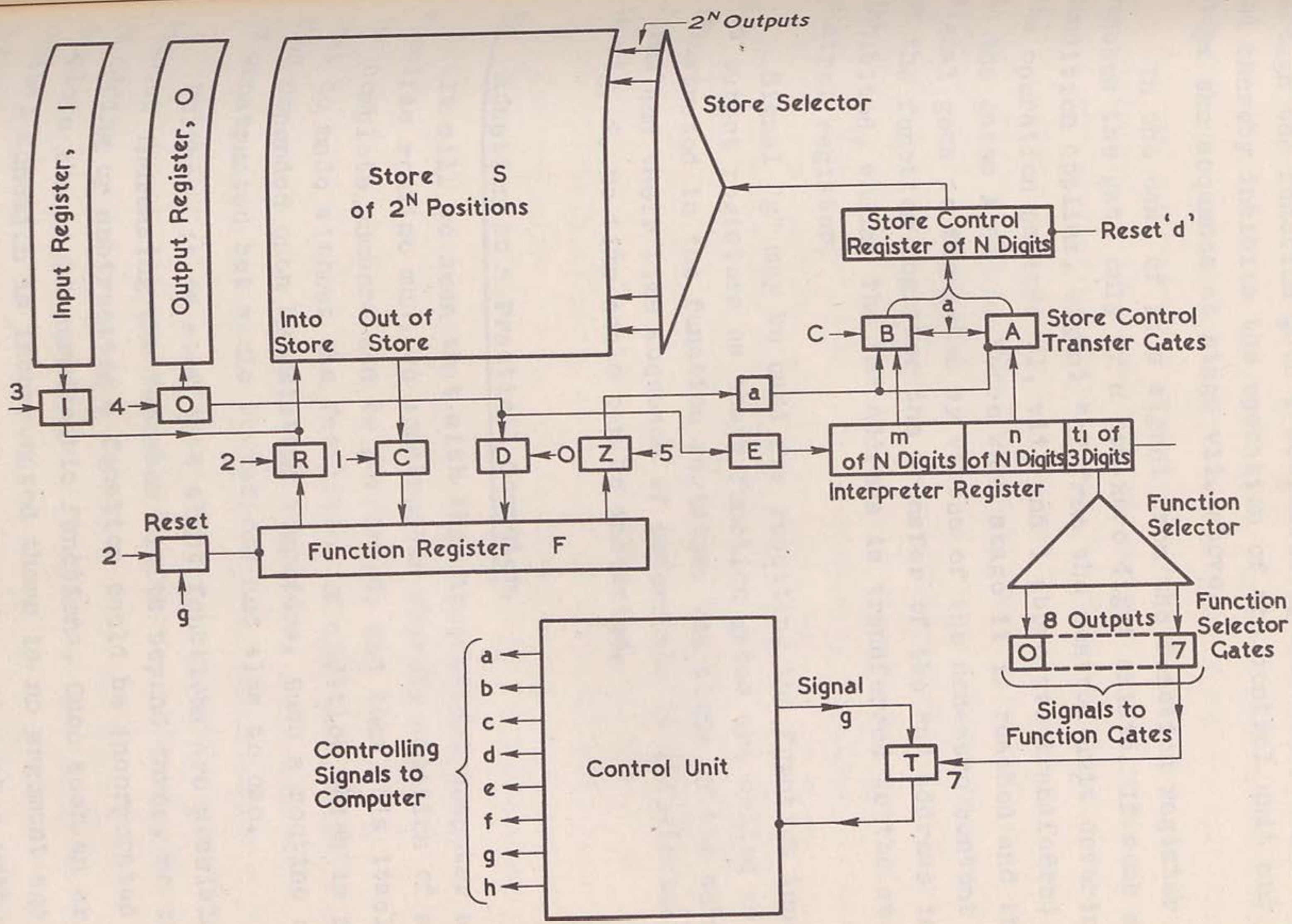


FIG. I. BLOCK SCHEMATIC OF THEORETICAL COMPUTER.

[illegible]).

The control [illegible] produces signals [illegible] which [illegible] of standard [illegible] controlling [illegible].

The sequence of functions [illegible] follows, commencing from the point [illegible] of [illegible] operation [illegible] satisfying the last command:

i. [illegible] the signal [illegible] or [illegible] respectively [illegible] control [illegible] in [illegible] the store register [illegible] the [illegible] "n" in the [illegible]

[illegible] be held in the store control [illegible] by the [illegible]

[illegible] as that the [illegible] is transferred to [illegible]

[illegible] input [illegible] via [illegible] store control.

vi. The signal [illegible] function unit [illegible]

[illegible]

vii. The store control [illegible] reset by [illegible] the sequence [illegible] cycle of operations.

[illegible] functions [illegible]

[illegible]

through the function gate T activated by function output 7 and thereby inhibits the operation of the control unit and stops the sequence at stage vii. above.

In the case of Z; a signal from the function register reaches the gate only if a non-zero digit exists. If such a condition applies, signal "h" from the control unit covering the operation periods vi, vii, and i above are transferred to the gates A and B. Hence when stage ii is reached and this signal gets to the gates by virtue of the non-zero content of the function register the transfer of the "n" address is inhibited, whilst the "m" address is transferred to the store control register.

Signal "g" may be used for resetting the function input and output registers as their function gates are called as illustrated in the function register. The stops of the wave-forms and their time sequence of occurrence is illustrated in fig. 2. No time scale can be indicated.

6. Adaption to a Practical Computer.

It will be seen that with this theoretical computer a specific routine must be constructed whereby addition of any two complete numbers can be performed; and that this itself must be made without the facilities of addition which is so much depended upon by existing computers. Such a routine can be constructed but would be tedious and slow to use.

Further, it is seen that eight functions are possible without increasing the f-number digits beyond three, so that on adding or subtracting a function could be incorporated to complete the eight permissible functions. Once such an arith-metical function is incorporated there is no argument against a sequential method of storing commands, and such a method greatly simplifies the command code, reducing it to a one-address system. A special counting "sequence register" must be provided however. This register would receive a unit digit count during each performance of the computer routine. Further, the store control register may be dispensed with so long as the sequence control register may be connected to the store selector during periods i, ii and iii, while the interpreter address is connected to the store selector, during the periods v, vi and vii. The Z function could, if addition or subtraction is incorporated, be more usefully converted to a test for negative sign, and if this function is called, and the contents of the function register is negative, the interpreter address is transferred to the sequence register, otherwise not. This corresponds to the E function of EDSAC. These adaptations illustrated by fig. 3, the same notation being used for controlling signals.

So far as the choice between addition and subtraction is concerned, subtraction should be given preference, since an addition can be constructed out of two subtractions, whilst a subtraction cannot be formed from additions.

The need for further functions will be discussed later.

7. Numerical Scale

The numerical scale usually adopted is the straight binary system in which subject data digits are given weightings of integral powers of two. Command addresses are those usually expressed each by a similar convention.

The binary scale is adopted, firstly because of its simplicity in its arithmetical laws, and secondly because it is the scale which is most economical in equipment, although the length of a binary number is the greatest possible, being about 3 times longer than an equivalent

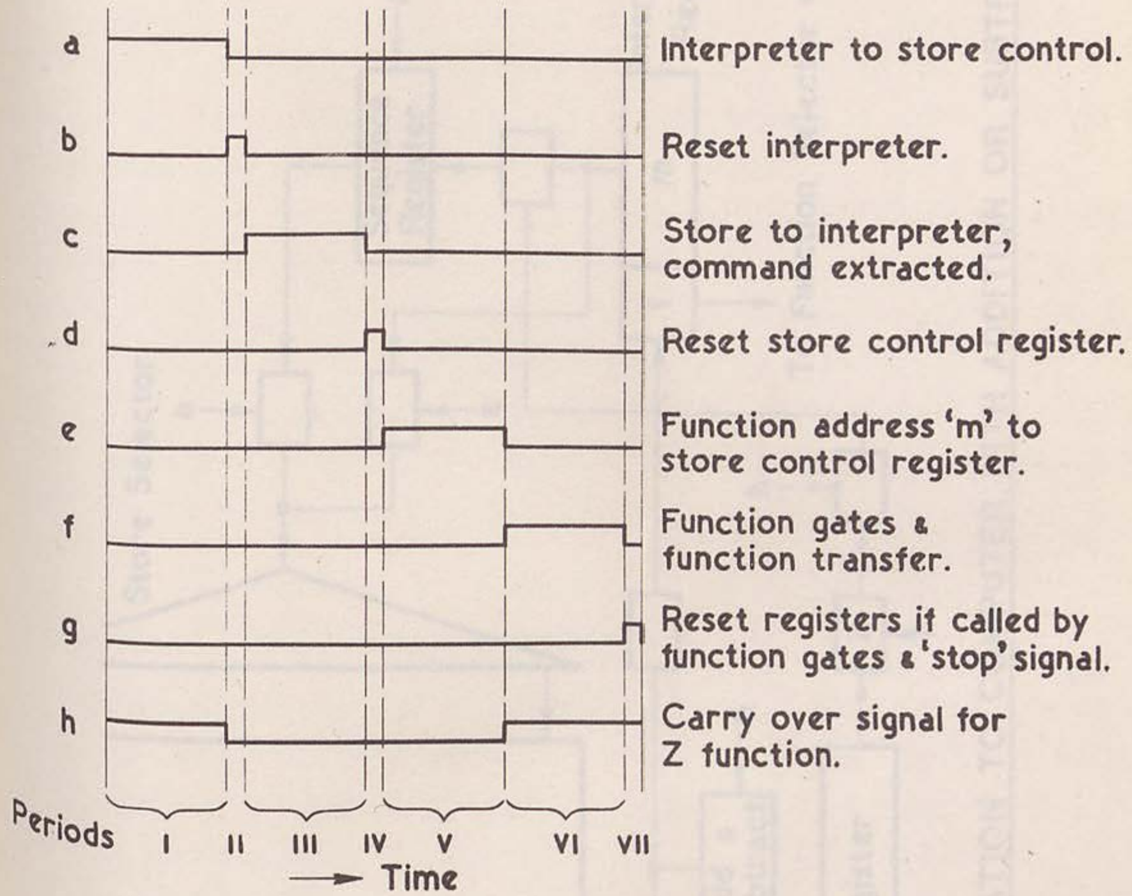


FIG. 2. CONTROL SIGNALS.

address system. A special counting "sequence register" must be provided however. This register would receive a unit digit count during each performance of the computer routine. Further, the store control register may be dispensed with so long as the sequence control register may be connected to the store selector during periods i, ii and iii, while the interpreter address is connected to the store selector during periods v and vii. The a function code, if addition or subtraction is incorporated be more and if this function is called, and the contents of the function the interpreter address connections to the sequence register, otherwise not. illustrated in Fig. 2, the pulse being used for

7. Numerical Basis

The numerical scale usually adopted is the binary scale. ... with two, compared ... by a similar argument.

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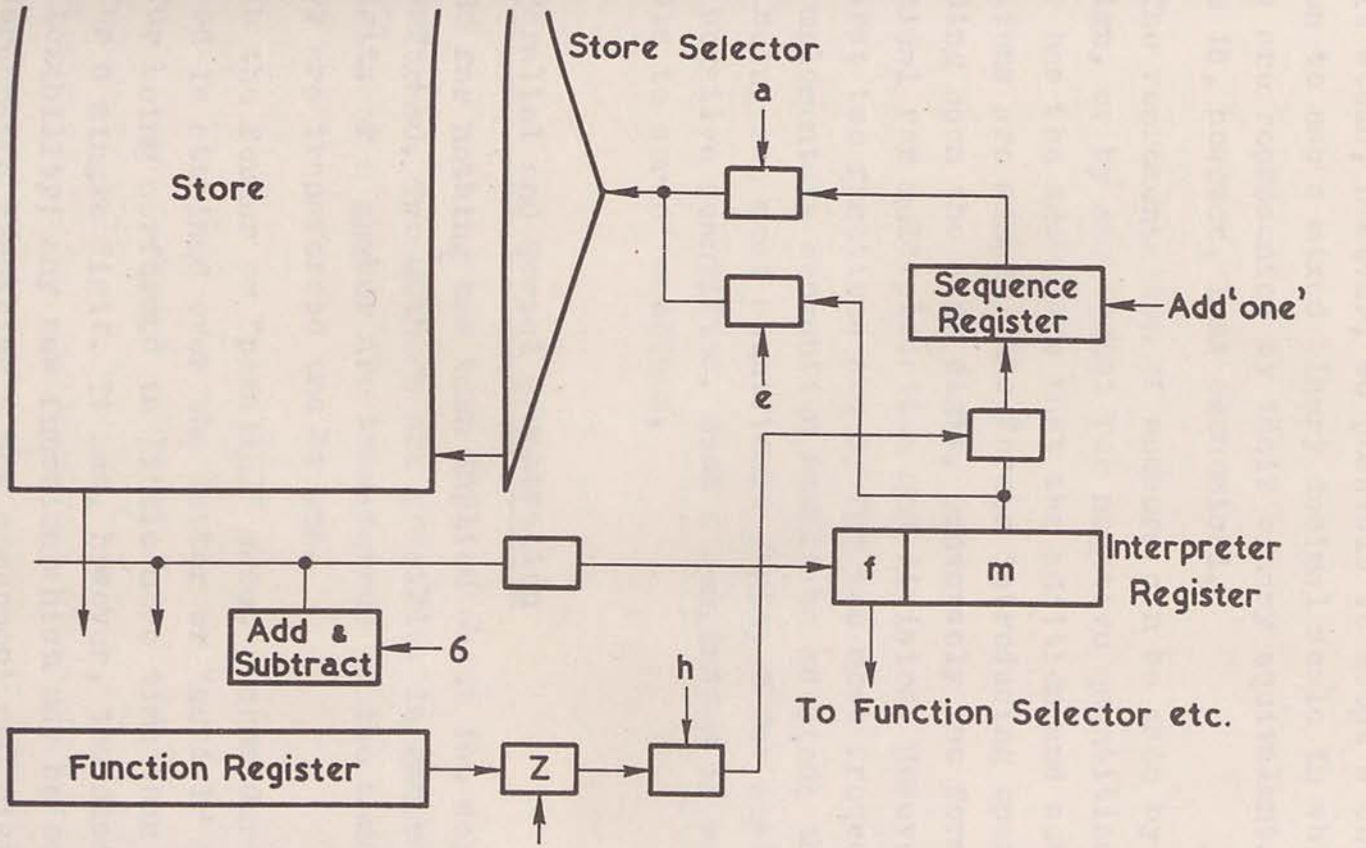


FIG. 3. ADAPTATION TO COMPUTER WITH ADDITION OR SUBTRACTION.

decimal number.

It would, however, be possible to adopt a decimal scale, or even to use a mixed binary decimal scale in which decimal digits are represented by their binary equivalent. Such a system is, however, less economical.

The representation of numbers can be made by modulus and sign, or by complement for negative quantities. The latter has the advantage that the addition and subtraction operations are simple, the former introducing operations depending upon the sign digit. Conversely the former is more economical for multiplication and division. However, since the first two functions are by far the most frequently used, the complementary convention should be adopted; the sign digit being the most significant digit, 0 for positive and 1 for negative quantities. Such a convention is particularly suitable to serial machines.

8. Parallel and Serial Organisation

So far nothing has been implied about the way transfers are performed. Two methods are possible, in general, either all digits of a number are transferred at the same time, or they are transferred one by one.

In the former or "parallel" case, a considerable increase in speed is attained over the latter or "serial" case; a transfer being performed in little more time than it takes to transfer a single digit. It has, however, the disadvantage of inflexibility; any new function which may be desired to be incorporated requiring a new component and digit channel for each digit position.

The serial system, although slower, possesses the advantage of requiring less equipment since all digits can be transmitted along a single channel, and also leads to a device of more flexible design than can be achieved with a parallel machine.

Another point distinguishing the two systems is that a large store and a very small group of functions is suitable for parallel devices; whilst in machines of limited store capacity programmes can be shortened by introducing more functions as is possible in a serial type machine.

9. Word Length and Address Systems

The number of digits per datum or "word" has so far remained unspecified. We would, however, desire to have a sufficiently large number of digits as well, providing sufficient accuracy for the great majority of calculators without the necessity to programme for the combining of two or more words together. A typical choice is an equivalent of ten decimal digits corresponding to 30 to 40 binary digits.

However, it appears that the average store capacity is of the order of 10^3 such sets of digits, although one would like to have an indefinitely large store. A command therefore of the one address type is likely to occupy between 13 - 17 digits only, and any increase in the number of addresses will increase this by about 10 digits for each extra address.

Thus if a 40 digit word length is adopted, it would be economical in digit capacity to adopt a three address system. However, practice in programming tends to show that a substantial number of commands would not use all three addresses thereby wasting capacity, and further it is believed that computer techniques are not yet ripe for a change to a multi-address system of such a size.

Another procedure is to adopt a one-address system which we have seen to be very convenient, and to allow two commands placed end to end to occupy the same space as a full number datum. If we adopt say 20 digits per command, we may adopt one of the following alternatives.

(a) retain a basic word length of 20 digits, and programme for joining words together

(b) to retain a 20 digit word and to transfer an extra 20 digit group according to the command control

(c) to adopt 40 digits as the basic word length for transfer and to place two commands into each store position

Of these alternatives course (a) has been adopted for the C.S.I.R.O. Mk I. machine, while the Cambridge EDSAC uses the course (b).

10. Special Functions

So far only the functions of addition and subtraction of the true arithmetical types of operation have been discussed. However, although multiplication can be programmed as a sequence of additions, it is common to supply a separate multiplying unit.

The function of multiplying in binary scale is not difficult to mechanise and is considered worth the effort and extra "hardware". There is a definite point in providing a serial machine with a limited store capacity with a multiplier, although in parallel machines using large stores, its inclusion may be of doubtful economy.

Division is so little required that dividing units are usually not incorporated. There are further arguments against such devices; first, division can be performed by multiplication by the reciprocal of the divisor, and a reciprocal itself can be determined by a series of multiplications. Further, division is complicated by the binary point convention, already fixed if a multiplier is installed, and by the relative size of divisor and dividend. Thus, first the divisor must not be zero, otherwise division is meaningless; second, the quotient must be within the register capacity. For instance, in machines which deal with numbers lying between 1 and -1, the divisor must be greater than the dividend prior to performing the division. All these conditions must be satisfied, and it therefore seems preferable to programme the tests, following which the division itself may as well be programmed also. Similar arguments apply to square-rooting devices.

Regarding the use of a multiplicity of function registers and sign-discriminating functions, no decision can be made at this stage. It may be found worth while to simplify programming by this means; and may eventually be convenient to make the entire store capable of adding and subtracting and so on. On the other hand, it may be found better to simplify machine design, and place all the responsibility for operation upon the programme design.

Discussion

D. R. HARTREE - in opening the discussion, referred to the simple machine described by Mr. Pearcey and said that addition being such a simple and basic operation in numerical work might well be regarded as the starting point in functional design.

Professor Hartree then commented on the relative merits of serial and parallel machines and said that reliability and ease of maintenance did not depend so much on the total amount of equipment in a machine as on minimising the number of different kinds of units. Parallel machines use a multiplicity of similar units and maintenance could therefore be reduced to a fairly simple procedure. The engineers at Cambridge were considering parallel operation for the Mk II machine.

Commenting on the use of a small high speed store in conjunction with a large auxiliary store he said that whilst it was functionally quite a sound idea it would complicate programming. He also mentioned "optimum programming". With a delay line store, time is wasted between one completion of an operation and the appearance of the next instruction. With optimum programming this is minimised but it is difficult to do with a one address type of machine. The A.C.E. at N.P.L. is designed to facilitate the use of optimum programming. This would be used for sub-routines, and for groups of operations which would be repeated many times in the course of a calculation, for which the saving of time would offset the increased complexity of programming.

T. PEARCEY - in replying to a question by Professor Cherry, who asked whether the increase in speed made possible by the new machines made complicated programming for still higher speed worth while, said that he did not think so at present.

D. R. HARTREE - supported Mr. Pearcey's view, saying that reliability if the principal factor determining the amount of work that a single machine can accomplish at present. In reply to a further question on the loss of digits in transfer operations, he said that Raytheon have proposed a scheme for forming a "weighted sum" of the digits before and after transfer of a number.

J. C. STEWART - explained a simple check system that had been included in the Bell System Relay Computer. The digit "0" is carried along with the number if the sum of its digits is even and a "1" if odd. Alteration of a single digit may then be detected. A system has been worked out using extra digits for determining where the error has occurred. These digit groups have been called "error correcting codes".

C. B. SPEEDY - said that the first system described by Mr. Stewart is a simplification of the weighted count system. The completeness of a checking system is related to the redundancy present in the representation of numbers. It is possible to approach an ideal system by using sufficient digits. The weighted count system checks all arithmetical operations except division.

W. R. BLUNDEN - indicated that the main factors to be considered in the design of a modern high speed digital computer are

(a) choice of numeral scale (binary or decimal)
(b) mode of transfer of data inside the machine (serial or parallel)
(c) address code
(d) type and capacity of the store

The following table gives the results of a recent survey covering nine machines in operation or under construction in the United Kingdom and the C.S.I.R.O. Mk I computer. It shows clearly that a great diversity of opinion exists, when specifying a computer, as to the best choice of the alternatives offered above.

Machine	(a)	(b)	(c)	(d)
Ferranti	binary	serial	single	electrostatic and magnetic drum
EDSAC	binary	serial	single	accoustic delay line
ACE	binary	serial	double	accoustic delay line
Harwell	decimal	parallel	double	scale of ten counting valves
TRE	binary	parallel	single	electrostatic and magnetic drum
Dollis Hill	binary	serial	quadruple	delay line
Birbeck College	binary	serial	double	magnetic drum
J. Lyons & Co.	binary	serial	single	accoustic delay line
RAESCC	decimal	parallel	triple	magnetic drum
CSIRO Mk I	binary	serial	double	accoustic delay line and magnetic drum

X. Some new developments in equipment for high speed digital machines

by D. M. Myers, D. L. Hollway, C. B. Speedy and B. F. Cooper

D. M. Myers: Introduction.

We have now discussed the various procedures necessary for solving a mathematical problem with the aid of an automatic digital machine, and we have seen something of the organization of such a machine from the point of view of the functional significance of its various components. The picture would not be complete without at least a passing mention of some of the techniques employed in carrying out the arithmetical and logical functions involved.

Some of these techniques have already been described at this conference, and more detailed accounts are available in references already given. In particular, the electronic devices which have been successfully used for storage of numbers and instructions are described in considerable technical detail. We propose therefore to confine our remarks to several developments in which we have been personally concerned and which will therefore be available for your examination during the course of the conference, in their various stages of completion.

2. Magnetic switching

In any type of storage device, there exists the problem of gaining rapid access to the contents of a specified storage location. This requires the use of a form of switch which will provide an electrical path between that location and the particular source or destination of the word which has to be transferred to or from the store. The switch should clearly operate at such a speed that the time lost in switching is rather less than the interval between clock pulses, and for this reason electronic switching has been generally preferred.

However, subject to achieving sufficiently rapid operation, there are advantages to be gained by the use of magnetic elements for switching, and an investigation has been carried out on the use of saturable magnetic cores. A core made of an alloy with suitable characteristics may be brought to magnetic saturation by a relatively small magnetising force. The effective inductance of a coil wound on the core will be very much less in the saturated than in the unsaturated state of the core. Thus the passage of direct current in a magnetizing coil may be used to reduce the impedance of another coil to a varying current, such as a current pulse, passing through it, assuming that the amplitude of the pulse is insufficient to affect the saturation of the core. The whole device may therefore be used as a switch, operated by the application of a D.C. potential to the magnetizing coil. The effectiveness of the switch depends on the ratio of unsaturated to saturated inductance, and its speed of operation is determined by the time constant of the circuit containing the magnetizing winding.

It is possible in a grain-oriented nickel-iron alloy, by careful attention to the purity of components and to heat-treatment, to achieve saturation at about 1,6 weber/sq.m. with a magnetizing force of less than 20 amp turns/metre.

It may be readily shown that the time constant of the magnetizing winding depends, inter alia, on the volume of material in the core, which must be made very small if reasonably high switching speeds are required. Experimental work has not yet progressed far enough to indicate the limiting switching speed attainable in practice, but it appears that the design of a magnetic switch suitable for use with a magnetic store having a pulse repetition frequency of the order of 10 kc/s would be practicable. The experimental results obtained so far indicate that further development will be required before the switching speed can be made sufficiently high to deal with pulse repetition frequencies of the order of 1 Mc/s, such as occur in certain electronic storage devices.

The advantage of a magnetic switch of the type described lies in the fact that a single magnetizing winding may be used to control a core containing a large number of pulse windings. This allows the use of a "matrix" system of switching, in which a group of n cores, each provided with one magnetizing winding and n pulse windings, can provide selective switching for n^2 storage locations. In the more conventional "radix" or tree system of switching, the same number (n^2) of storage locations would require ($n^2 - 1$) bi-stable electronic elements. This comparison is of course approximate but illustrates the principle involved.

In comparing the two systems, it is necessary to take into account also their relationship to the techniques employed generally in the machine. It may well be that the advantage of the magnetic system in regard to component requirements and simplicity is only an apparent one, and no valid assessment of its merits can be made until it has been applied and tested in full operation. However, there are obvious advantages in a switching system which can be sealed in a single container and which should require little maintenance, and it is hoped to develop the system more fully in the near future.

3. Switching, gating and counting

The control and arithmetic units of a calculating machine depend on the use of components which allow a number or instruction, expressed usually as a group of electrical pulses, to pass from a prescribed source to a prescribed destination. An electronic "gate" is widely used for this purpose; the gate is opened or closed to the passage of incoming pulses according to the presence or absence of an electrical "gating" potential. There are several well-known electronic devices for this purpose.

The gating potential is usually provided by a bi-stable device, such as a multi-vibrator, actuated in the first instance by a pulse, which is the signal to commence carrying out the instruction. On receiving the pulse, the bi-stable element provides a gating potential, which holds the appropriate gate open until, at the conclusion of the operation, a further pulse applied to the bi-stable element removes the gating potential and so closes the gate. Although this procedure may be varied according to the organization of the machine, it is in very general use in existing and projected machines.

The bi-stable element and gate together form a functional unit; in most modern machines there are many such units each of which, in its conventional form, contains a considerable number of circuit components. It was thought worthwhile to consider the possibility of combining the whole of the functional operations of the bi-stable element and gate, so that they may be carried out in a single vacuum tube, which Mr. C. B. Speedy will describe to you this morning. This tube, at present in the experimental stage of development, is intended as a substitute for the conventional circuits associated with switching and gating operations.

The device follows an electronic beam tube which Mr. D. L. Hollway has developed for decimal counting and will describe soon. This tube, which was designed in the first place for an entirely different purpose, will have limited significance in a machine which operates in the binary scale, but in association with the input and output mechanisms of such a machine, or in a decimal machine it may have advantages over existing circuits for counting or switching.

4. Magnetic storage

The demonstration of the electronic calculating machine in the Division of Radiophysics provided an opportunity for seeing in operation a storage system based on the use of acoustic delay lines. In association with this, an auxiliary store will be used to hold function tables and other data which may be called on from time to time for transfer to the main store. The auxiliary store is in the form of a magnetic drum, which has required considerable ingenuity in its design and in its interconnection with the machine itself. Mr. B. F. Cooper will give you a brief description of this device.

D. L. Hollway: A Beam Tube for Decimal Counting

As a contrast to the complex questions with which this conference has occupied itself during the past two days, the device to be described is, in principle, very simple. It is a single electron tube scale-of-ten counter which has been developed for general use in counters, chronographs frequency dividers and so on. A slightly different form of the tube would be suitable also for some applications in electronic computing equipment.

The counter is an electrostatic high vacuum tube in which counting is accomplished by successive displacements of a deflected electron beam through a number of stable positions [27]. The operating principle may be explained by considering the simplified electrode arrangement shown in fig. 1.

An electron beam enters, along the axis, a system of five similar deflector plates maintained near the potential of the final anode by five equal series resistances. The beam forms a focussed spot in the plane of the collecting segments, each one of which is directly connected to the correspondingly lettered deflector.

If one deflector, B, is lowered momentarily in potential, an electrostatic field is formed which deflects the beam away from plate B and moves the spot in the direction of the arrow B'. The beam current which then flows to collector B will lower the potential of the collector, and hence of the deflector, sufficiently to maintain the deflection. A similar stable condition is possible at each of the other collectors, after initial deflection. Now imagine the deflector plates to be turned about the axis one tenth of a revolution clockwise. Deflection of the spot in the new direction B" will cause part of the beam current to flow to collector C, which in turn moves the spot further across plate C. There are no stable regions and the spot rotates continuously in the

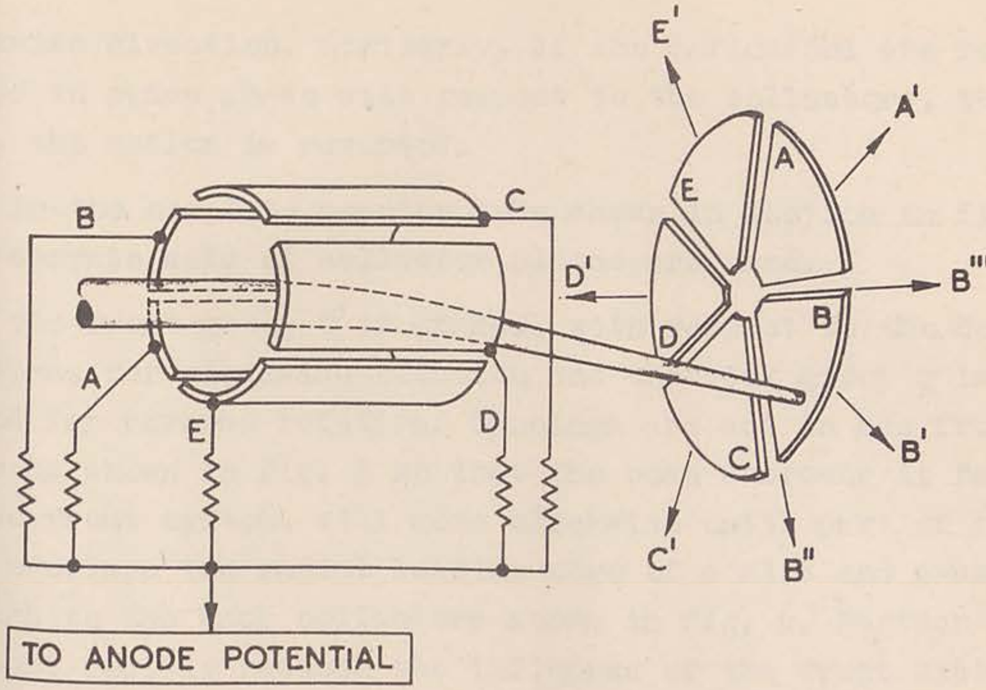


Fig.1 - Simplified arrangement of counter electrodes

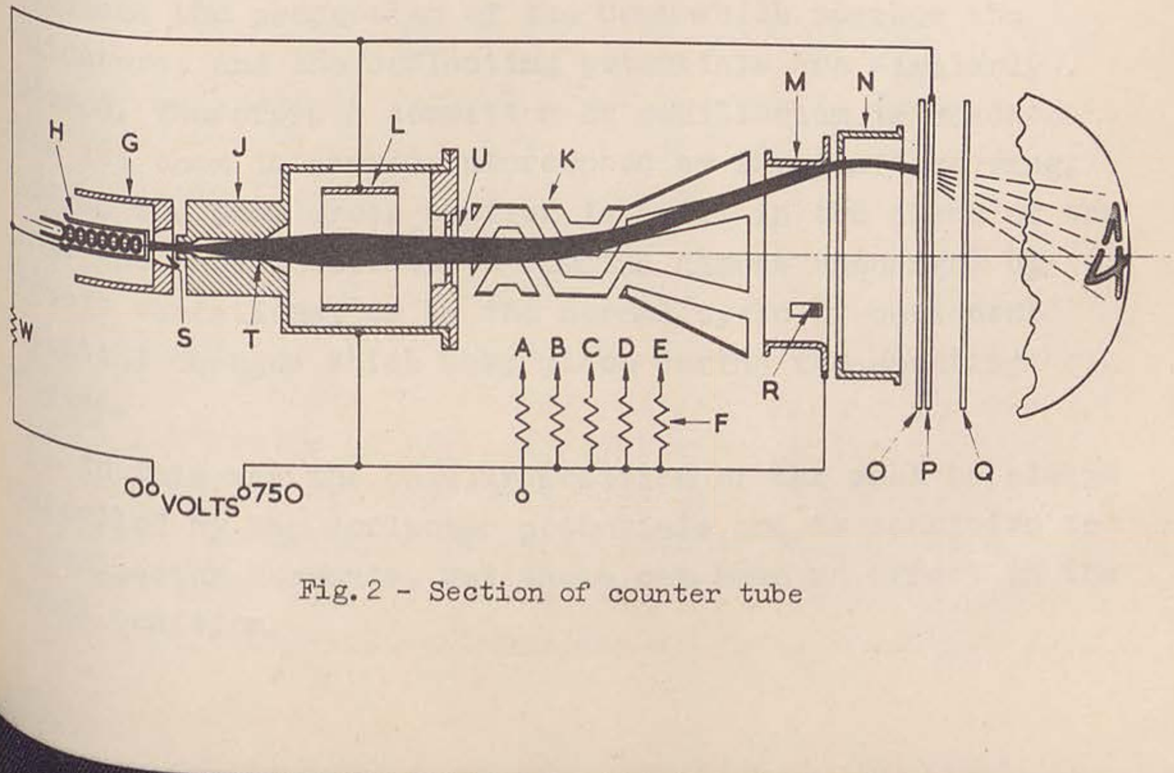


Fig. 2 - Section of counter tube

clockwise direction. Similarly, if the deflectors are retarded in space phase with respect to the collectors, to B''', the motion is reversed.

In the complete counter tube shown in section in fig. 2, two separate sets of collector plates are used.

The front group O is phased, with respect to the deflectors, for clockwise rotation and the back group Q is phased for reverse rotation. Openings are cut in the front group as shown in fig. 3 so that the beam wherever it falls on the front system, will move clockwise until part of the spot overlaps the radial leading edge of a slot and passes through to the back collectors shown in fig. 4. Further movement rapidly reduces the influence of the front system and increases that of the back, so that a state of angular equilibrium is reached, and the resultant deflection is purely radial. Therefore the beam moves outwards until it is partially intercepted by the inside edge of the ring M, fig. 2, the "positive ring", which is maintained at the potential of the final anode. Any further radial deflection decreases the proportion of the beam which reaches the collectors, and the deflecting potentials are similarly reduced. Therefore a condition of equilibrium is reached when the beam is partly intercepted by the positive ring. Because the beam cross section is small in the plane of the ring, the radial deflection remains almost unchanged by circuit variations, or by the normal cycle of deflector potential changes which take place during the counting process.

In this way the angular position of the spot is always controlled by the deflector potentials and is sensitive to the collector currents, but these can have no effect on the radial position.

The radial position is controlled only by the trigger electrode, N, fig. 2, a short cylinder which receives the signals to be counted.

When a positive going signal appears, the spot moves radially outwards from the position "1", fig. 3, until the outer edge of the slot is reached. Beyond this point the anti-clockwise restraint from the back collectors is reduced and the spot moves clockwise to the outer opening "$1\frac{1}{2}$" which is stable for this, and higher, trigger potentials. The beam is held in position "$1\frac{1}{2}$" until the removal of the input signal moves the spot inwards, once more on to a clockwise region, causing a further rotation to position "2".

Therefore each input signal moves the beam from one inner stable position to the next, increasing the stored count by one unit, and ten waves cause the spot to complete a full revolution and return to the initial position.

Because the intermediate states are stable, the input signals may be of any duration and amplitude above the minimal values necessary to register a count.

A second function of the trigger is the suppression of secondary emission from the front plates. Some interchange of secondaries is permitted between adjacent plates in both front and back systems in order to develop current waveforms corresponding to smoothly rotating deflection fields, but complete suppression of secondaries between the two groups of collectors, and between the collectors and the other electrodes, is essential. This is ensured by keeping the trigger at a lower potential than the collectors, and by inserting a suppressor (P, fig. 2) at cathode potential, between the front and back collectors. The suppressor has slots to match those in the front collectors. The outer

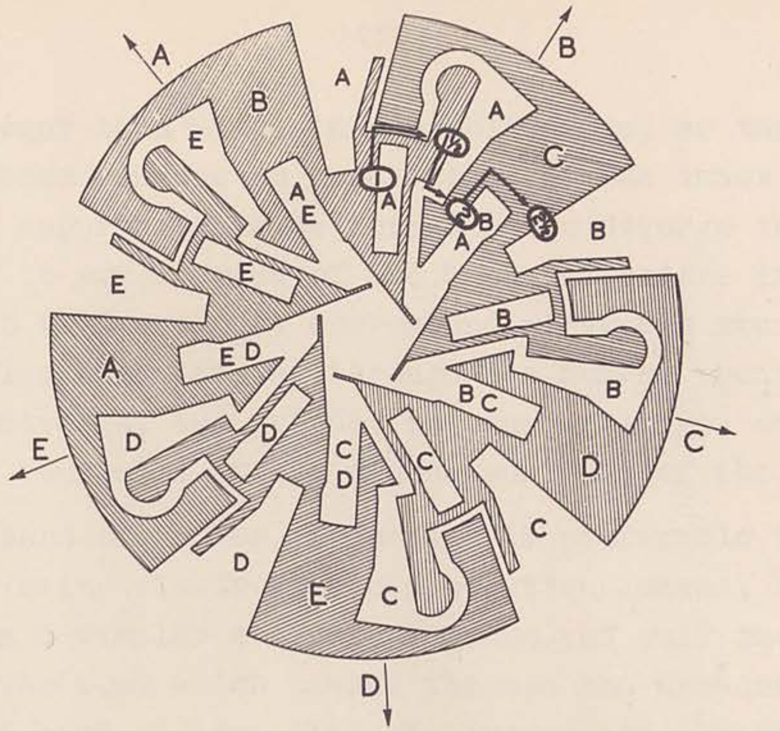


Fig. 3 - The front collector segments (The arrows show the direction of beam deflection for each of the five deflectors)

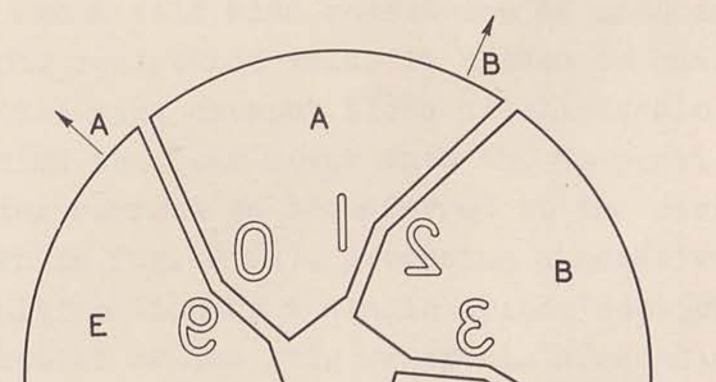


Fig. 4 - The back collector segments

group are larger than those in the collectors, so that the transmitted beam remains in good focus. At the inner openings, however, the suppressor is so formed as to diverge the beam sufficiently to cover areas of the back collectors in which the figures 0 to 9 are cut, corresponding to the count positions. Electrons passing through the number openings continue to diverge, and project the number image, enlarged eight times, centrally on the fluorescent end of the bulb.

This method of number projection is preferable to an earlier form using electrostatic projection lenses, because it results in a simpler collector system and only the small fraction of the beam which passes through the numbers is lost from the back plates. Also the images are always in sharp focus and satisfactory projection is maintained for a wide range of trigger potential.

In operation 750 volts is applied to the positive ring and anode and a self bias resistance is used in the cathode lead. The resistance value is chosen to ensure that some positive ring current flows at all stable states. At the tenth pulse the beam moves into the $9\frac{1}{2}$ position and the positive ring current is transferred to the carryover electrode, shown in fig. 2 (R), producing a negative pulse, which after amplification by a single triode section, raises the potential of the trigger of the succeeding counter in the chain, and increases its count by one unit. The usual "trigger" circuit needed with multiple tube counters operating from zero frequency upwards is not required, as the necessary discontinuity results from the form of the collector plates. Thus the changes occur at fixed potentials termed the upper and lower changeover points. At an anode potential of 750 volts, the trigger may rise above the lower changeover point at 120 volts far as 360 volts before the half count states become unstable. Similarly the count states are stable between the upper changeover, at 180 volts, and - 300 volts.

The counter is set to zero by dropping, momentarily, the potential of the appropriate deflector. Any even number may be set in this way, and any odd number by lowering an adjacent pair of deflectors.

In each state the beam is controlled by "error actuated" feedback to the deflectors, and by the limiting action of the positive ring. These controls compensate for normal inaccuracies in construction, variations in the collector resistances and changes in the applied voltages.

A complete counter, excluding the power supply, having four scale-of-ten stages is shown in fig. 5. Each tube is $4\frac{1}{2}$" long and $1\frac{1}{8}$" in diameter. At present the highest operating speed is approximately 100 kc/s.

The principle of beam switching between stable states may be extended to a number of tubes for special purposes. One of these is a reversible counter which will add or subtract signals from the stored count. Either the reversible form, or the normal counter, may be constructed with two or more trigger rings instead of one. This would allow signals form two or more sources to add or subtract from the count, while maintaining complete electrical isolation between the inputs. Coincident opposed signals would cancel and produce no change in the count.

By a modification to the collector system it would be possible to accept without error coincident positive signals from a pair of triggers, combinations of positive and negative, and coincident negative signals.

A scale-of-ten counter having ten plates instead of five, may be needed in applications requiring a separate

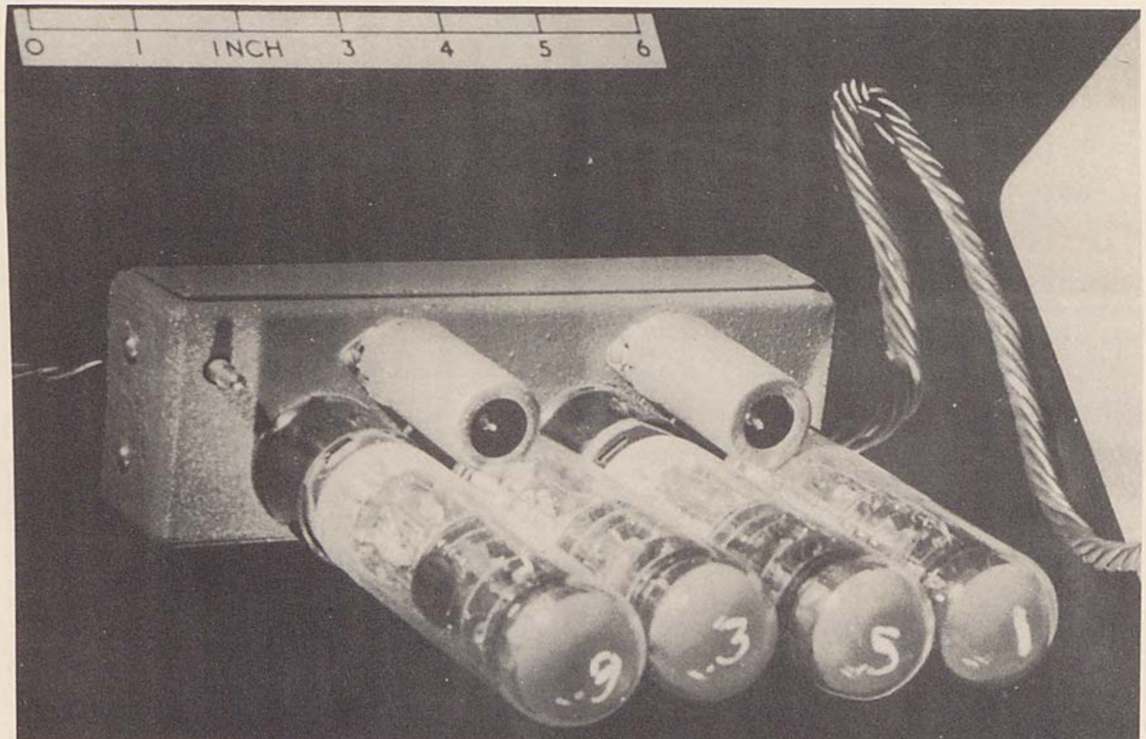


Fig.5 - Four decade scale-of-ten counter

output line for each digit. The complication of the five additional plates may be offset by the use of a different, simpler electrode structure, which however is not suitable for five plate counters.

Scale-of-N counters may be constructed, either by changing the number of stable states in each collector or by changing the number of collectors and deflectors. As N is reduced the allowable beam current and the highest counting speed rise, because of the increase in the size of the stable openings.

The counter forms part of the research program of the Division of Electrotechnology, C.S.I.R.O. The work was carried out in the C.S.I.R.O. Valve Laboratory. It is wished to record appreciation of the interest and encouragement of Professor D. M. Myers, contributions made by Mr. A. M. Thompson (C.S.I.R.O.) and the co-operation of the staff of the Valve Laboratory under Mr. R. E. Aitchison, of the University of Sydney.

C. B. Speedy: A Binary Gating Tube

A brief description is given of a vacuum tube which is at present under development at the Electrical Engineering Department of the University of Sydney. The tube is intended primarily for use in digital computing mechanisms where it performs a combined storage and switching function. The principle of the tube in its present state of development is described, although it is anticipated that changes in the electrode assembly and possibly in electrical principles will be found desirable as the work proceeds. It is however unlikely that the operational requirements will be modified. To indicate the manner in which it is anticipated that the tube will function in computing mechanism, mention is made of circuits for a number of standard computing components, (such as counters) using the proposed Binary Gating Tube.

It is observed that in a computing mechanism there are essentially only three basic elements. From an appropriate assemblage of these elements it appears possible to construct any computing or switching circuit at present built from passive circuit elements and conventional vacuum tubes. The three basic elements are:

	the bistable element	(storage element)
	the gate	(control element)
and	the diode	(one way element)

Pulse delay units are often used, however, since an equivalent result can be obtained using a combination of the bistable and gate elements, it is not considered here as a basic element. The object of the present work is to develop a single tube which combines the function of a bistable element and a pair of gates. Two pulse inputs to the bistable section are provided thus enabling the device to be set in a particular state. The tube is designed so that one gate is open and the other closed, depending upon the state of the bistable section.

In developing the tube it is appreciated that to be of any value in the computing field, it must compete with existing circuit elements. In the design of the tube, emphasis has been placed on the necessity for ease and simplicity of construction. The tube design in its present form consists of several electrodes of simple shape which could be manufactured by blanking and pressing operations. The operation of the tube is not critically dependent upon the electrode configuration, thus enabling wide manufacturing tolerances to be assigned to electrode size, shape and positioning.

The speed and reliability of the Binary Gating Tube appears, from the nature of its construction, to be similar with that of existing electron tubes using oxide cathodes. Because this tube can perform functions which normally require several conventional tubes, it is probable that the extensive use of the special tube would increase the overall reliability of a computing mechanism.

The use of the Binary Gating Tube has the advantage of simplifying the construction and maintenance of a computing system by reducing the number of different circuit components. It is anticipated that the bulk of the tubes in a computer would be Binary Gating Tubes and diodes, the remainder being heavy current tubes used as cathode followers. Each Binary Gating Tube requires in general, two resistors. The values are not critical and of the order of several megohms. These resistors are connected to the input deflecting plates of the tube. They prevent the potential of these electrodes from "floating" when they are connected to an open gate of another tube. The cathode followers would require a cathode load resistor, and the diodes would require no resistors. Because of the beam deflection principle used in the proposed tube, direct connections between the output of one tube and the input to another is possible. As a result, capacitors would not be required at any point in the circuit. Hence circuits using the special tube would consist of only five component items:

binary gating tubes
diodes tubes
heavy current tubes

resistors of the value approximately 5 megohms
cathode resistors of approximately 20,000 ohms

The ratio of resistors to tubes would be of the order of 2:1.

2. Principle of Operation

The form of the Binary Gating Tube which appears at present to be most promising is shown in fig. 1. The drawing is not to scale, and the voltages shown are approximate values. The electron gun is of the Pierce type in which the emission is space charge limited. Two sets of X-deflectors are shown. The pair nearer the gun are brought out to pins in the base, while the second set are concerned with the maintenance of the stable states. Four collectors and four targets are also shown. Two of these are part of the bi-stable section of the tube. The remaining collectors and targets form gates denoted by Gate A and Gate B.

The state of the tube depends upon the location of the electron beam in the tube. The geometry of the tube is such that the beam may strike either the left or the right hand targets depending upon the particular state in which the tube is existing. As shown in the diagram the left hand targets are being bombarded. The two front targets (those nearer the gun) are connected to one another and to the right hand deflector plate of the second pair of deflectors. There are no other electrical connections to these targets in which case they are electrically floating. The targets are good secondary emitters, and the potential to which they adjust themselves is that of the electrode which is collecting their secondary electrons. As shown, this system adjusts itself to the potential of 400 volts. Since the

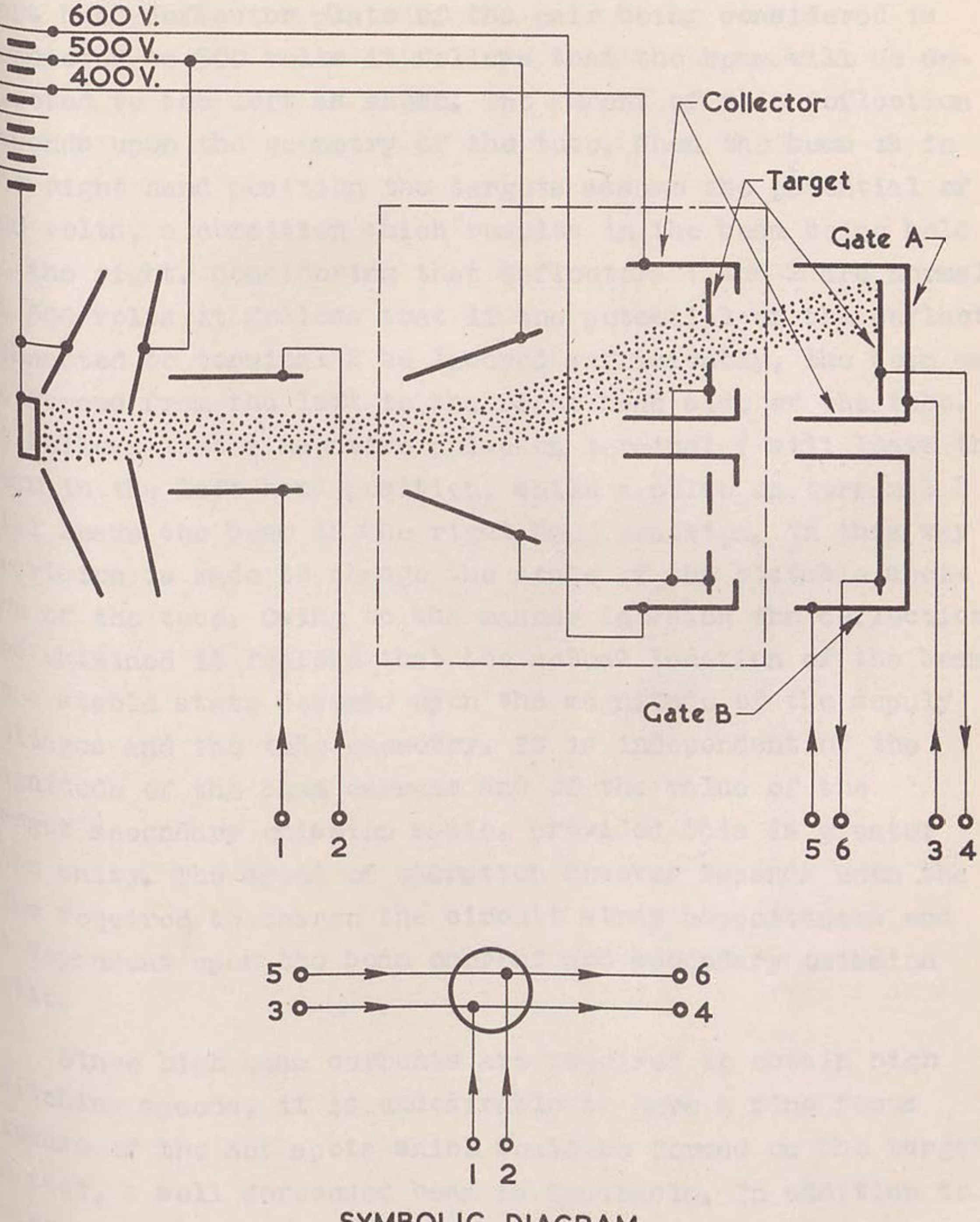


FIG. I. BINARY GATING TUBE.

left hand deflector plate of the pair being considered is connected to 500 volts it follows that the beam will be deflected to the left as shown. The amount of this deflection depends upon the geometry of the tube. When the beam is in the right hand position the targets assume the potential of 600 volts, a condition which results in the beam being held to the right. Considering that deflectors 1 and 2 are normally at 500 volts it follows that if the potential of the deflector connected to terminal 2 be lowered sufficiently, the beam may be forced from the left to the right hand side of the tube. It follows that a negative pulse on terminal 1 will leave the beam in the left hand position, while a pulse on terminal 2 will leave the beam in the right hand position. In this way provision is made to change the state of the bistable section of the tube. Owing to the manner in which the deflections are obtained it follows that the actual location of the beam in a stable state depends upon the magnitude of the supply voltages and the tube geometry. It is independent of the magnitude of the beam current and of the value of the target secondary emission ratio, provided this is greater than unity. The speed of operation however depends upon the time required to charge the circuit stray capacitances and is dependent upon the beam current and secondary emission ratio.

Since high beam currents are required to obtain high switching speeds, it is undesirable to have a fine focus because of the hot spots which would be formed on the targets. In fact, a well defocused beam is desirable. In addition to decreasing the beam current density, a large diameter beam has an additional advantage. The location of the beam is affected by stray magnetic fields, and if the beam diameter is made large the misalignment of the beam is made relatively small.

Two gates are shown, these are referred to as Gate A and Gate B respectively. Each gate consists of a collector and a target. Portion of the beam striking the front target passes through a slot to the gate target. This target under bombardment assumes the potential of the collector surrounding it. If a negative pulse be applied to this collector, a similar pulse will appear on the target. Under this condition the gate is "open"; a pulse applied to its input appears at its output. Gate B under this condition is "closed". The potential of this target is independent of the potential at terminal 5.

To make connections to the tube an 11 pin base is required. Three power supply connections are required and these are 400, 500 and 600 volts approximately.

Visual indication of the state of the tube is available by allowing portion of the beam to pass through the gate targets and to strike a fluorescent screen.

3. Applications

Nomenclature

Denote a storage element by S.

Denote the contents of S by C(S),

$C(S) = 0$ or 1

Digit representation. Digits are transmitted by means of a Digit Bus consisting of two lines referred to as the D and the 1/D lines respectively.

Digit 1 is represented by a pulse on the D line
and no pulse on the 1/D line.

Digit 0 is represented by a pulse on the 1/D line
and no pulse on the D line

In the absence of a digit no pulse appears on either the

D or the 1/D lines.

Note: This method of pulse representation of the digits is not essential to the operation of the Binary Gating Tube but is found to simplify the circuits.

Clock period. This is the fundamental interval of time in the computing circuits. Denote this by T.

Clock pulse. These are pulses generated in the computing mechanism at times n.T, where n = 0, 1, 2, ... etc. Denote by pc.

Shift pulses. Occurrence times $(n + \frac{1}{2}).T$
Denote by ps.

Quarter pulses. Occurrence times $(n + \frac{1}{4}).T$
Denote by pd.

Three-quarter pulses. Occurrence times $(n + \frac{3}{4}).T$
Denote by pb.

Counters. Denote input pulses by pi.
Denote output pulses by po.
Denote the number of input pulses by (pi)
Denote the number of output pulses by (po)
An N-stage counter is defined by:

$$(pi) = N.(po)$$

Binary counter N = 2
Decade counter N = 10

Ring Counters

The general case of an N-stage counter is shown in fig. 2. Provision is made to clear it to zero by a clear-to-zero pulse at time n.T. The effect of this pulse is to set C(S1) to unity, and C(S2), C(S3), ...C(SN) to zero. The pulses pa are quarter pulses applied continuously at times

$(n + \frac{1}{4}).T$. Following the clear-to-zero pulse, the pa pulse sets C(A1) to unity and C(A2), C(A3), ... C(AN) to zero. An input pulse pi transfers C(A1) to S2. The following pa pulse transfers C(S2) to A2. Hence the result of a single pi pulse is to transfer C(A1), which is unity, to A2. The result of (N - 1) pi pulses is to transfer the C(A1), which is unity, to AN. The N-th pi pulse transfers C(AN), which is unity, to A1. In doing so an output pulse po is produced. In this circuit it is noted that 2N Binary Gating Tubes are required for an N-stage counter.

A binary counter can be derived from the N-stage counter of fig. 2 by making N = 2. This would require four Binary Gating Tubes. A simplified binary counter is shown in fig. 3 which requires two Tubes. Provision is made to enable the counter to be cleared to zero.

In fig. 4, a simplified decade counter is shown. It consists of twelve Tubes. The first two are connected as a binary counter stage. Pulses pa are applied continuously at times $(n + \frac{1}{4}).T$. Input pulses pi are applied to this binary counter stage. The output pulses from this stage are alternately applied to the lower and upper lines of the main counter. In clearing-to-zero, C(S1) is made unity and C(S3), C(S5), C(S7) and C(S9) are made zero. The binary stage is set by the clear-to-zero pulse so that the first pulse from it passes to the lower line of the main counter. In doing so it transfers C(S1) to S2
Subsequent pi pulses transfer C(S2) to S3
C(S3) to S4, etc.

The tenth pi pulse passes through S10 to form the output pulse po.

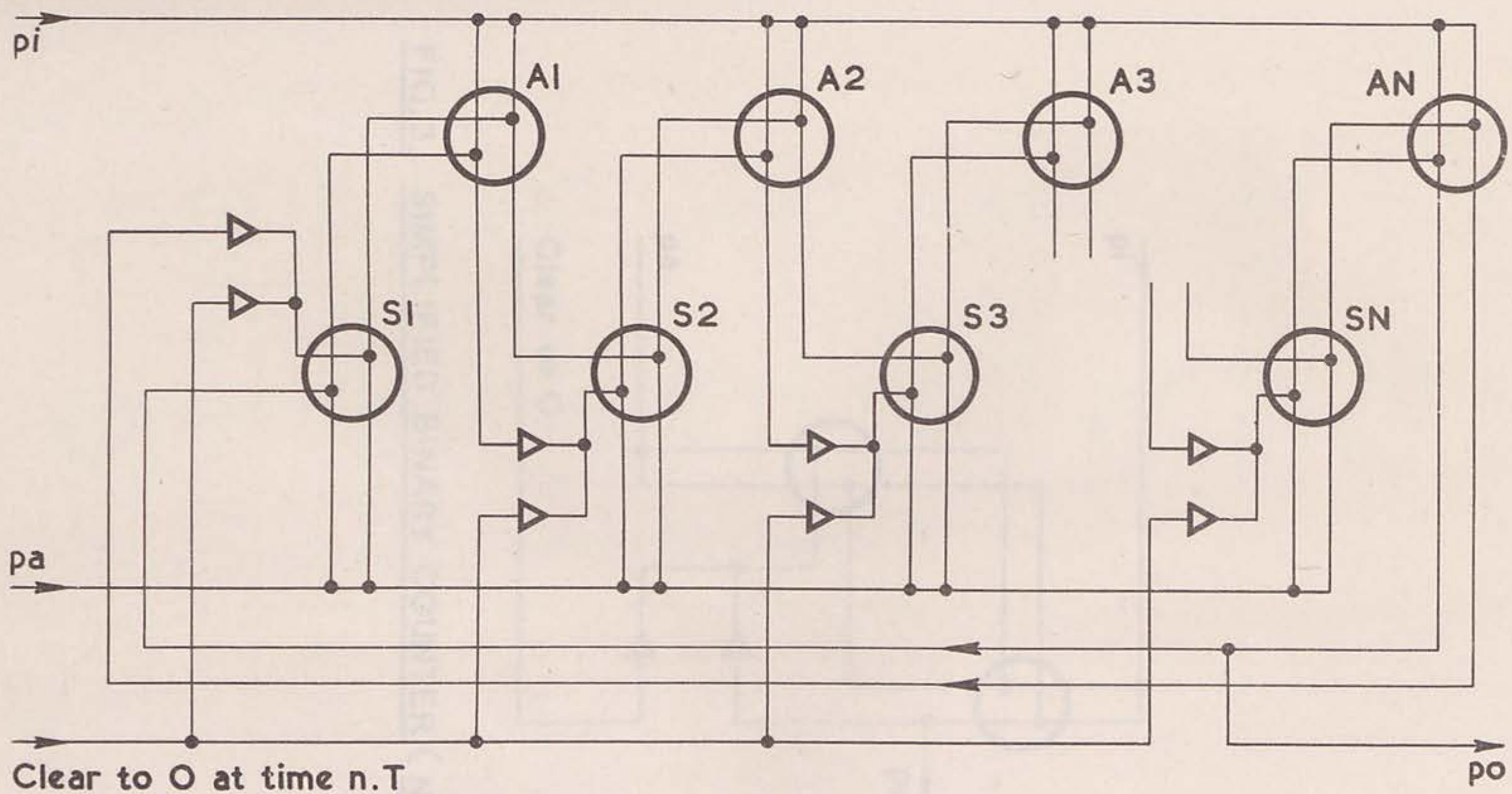


FIG. 2. N-STAGE COUNTER (GENERAL CASE).

(n + 1)T. Following the minus-to-zero pulse, the [illegible] pulse sets C(A1) to unity and C(A2), C(A3), [illegible] to zero. An input pulse p1 transfers C(A1) to [illegible]. The following p1 pulse transfers C(A2) to A3. Hence the [illegible] of a single p1 pulse is to transfer C(A1), which is [illegible] to A2. The result of (N - 1) p1 pulses is to transfer [illegible] C(A1), which is unity, to [illegible]. The N-th p1 pulse transfers C(AN), which is unity, to [illegible]. In doing so an output [illegible] p2 is produced. In this circuit it is found that [illegible] gating tubes are required for an N-stage counter.

A binary counter can be derived from the N-stage [illegible] of Fig. [illegible] by setting N = 2. This would require four [illegible] gating tubes. A simplified binary counter is shown by Fig. [illegible] and requires two tubes. Its operation is [illegible] [illegible] to be obvious [illegible] however.

[illegible] a simplified binary counter [illegible] [illegible] The [illegible] are connected [illegible] [illegible] supplied continuously [illegible] are applied to [illegible] [illegible] from this [illegible] [illegible] and output [illegible] [illegible] C(G1) [illegible] [illegible] are [illegible] [illegible] [illegible] [illegible] of the [illegible] [illegible] transfers C(E1) to [illegible] [illegible] C(E2) to [illegible] C(E3) to [illegible]

The [illegible] from the [illegible] [illegible]

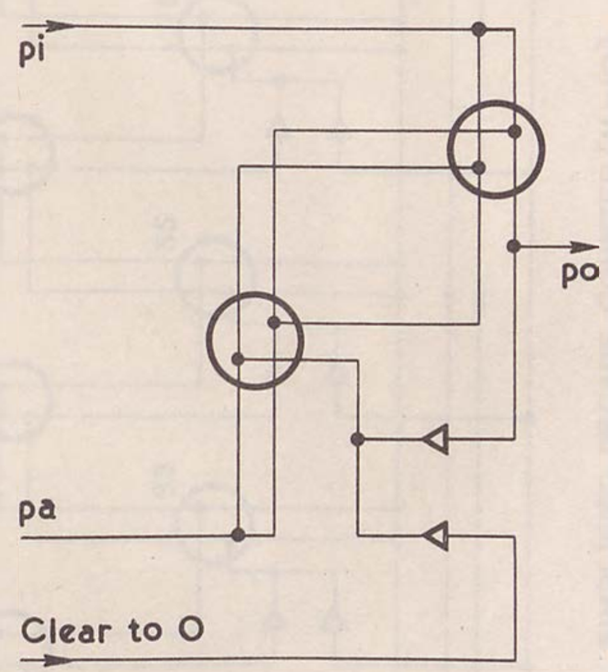


FIG. 3. SIMPLIFIED BINARY COUNTER (N = 2).

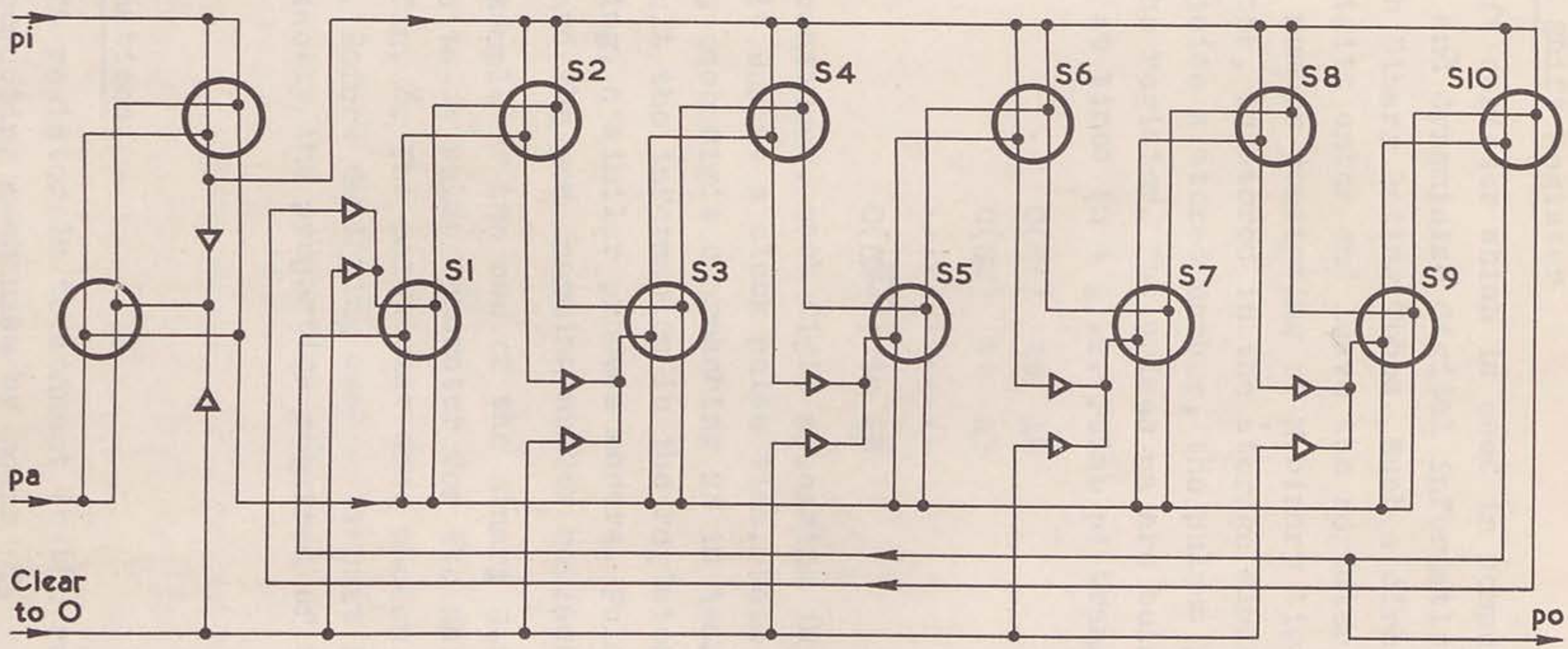


FIG. 4. SIMPLIFIED DECADE COUNTER (N = 10).

N-stage Shift Register

The shift register which is used in computer mechanisms to staticise and dynamisise digital information may be constructed with Binary Gating Tubes. Such a circuit is shown in fig. 5. Digits enter and leave the register via the Digit Bus. A number consisting of N binary digits, staticised by the register, is stored in the storage elements S1, S2, ... SN. To dynamicise a stored number, the pulses p1 and p2 are applied to the register. The pulses pa are being applied continuously at times $(n + \frac{1}{4}).T$. Pulse p1 transfers,

C(S1) to A2
C(S2) to A3
....
C($\overline{SN-1}$) to AN

The process continues, each digit appearing in succession upon the Digit Bus at a clock pulse time. Note that during this process, each digit on reaching SN is transferred to A1. As a result the information in the register is retained. In dynamicising, a similar process occurs. Pulses p2 are not applied and the new incoming number replaces the old one.

As an example of the use of the Binary Gating Tube in control circuits, a Pulse Generator for the Shift Register is shown in fig. 6. The particular case chosen is that in which N = 10. Before designing such a circuit it is necessary to specify exactly the properties required of the shift register.

Specifications

1. The shift register is to connect to the remainder of the computing mechanism by means of,

(a) Digit Bus. This bus, consisting of two lines

D and 1/D, transmits digit pulses either to or from the register.

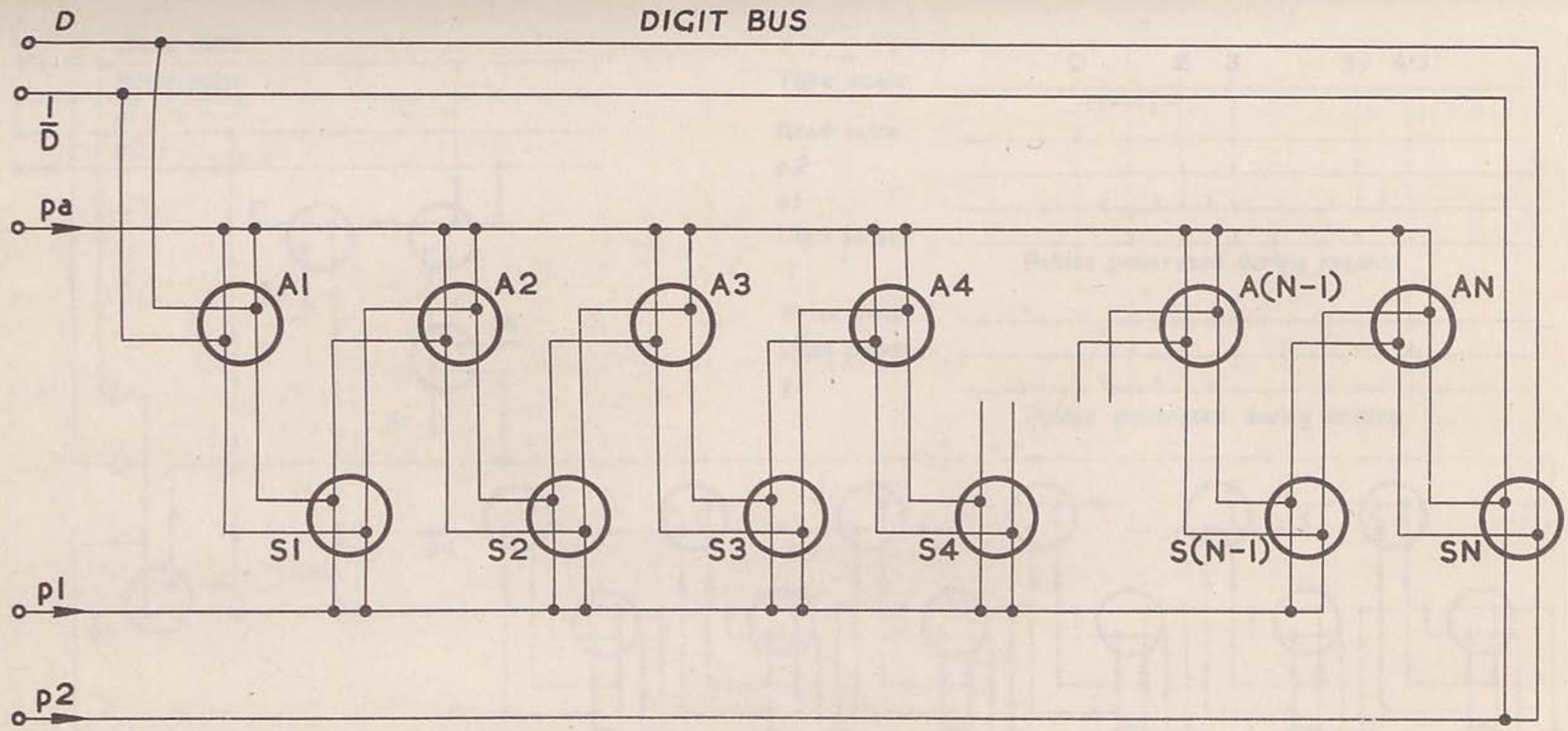
(b) Read Bus. A "read pulse" applied to this bus initiates the transfer of the N-digit number from the register to the Digit Bus.

(c) Write Bus. A "write Pulse" applied to this bus initiates the transfer of a number from the Digit Bus to the register.

B. Reading and Writing. During reading, the number in the register is to remain unaltered. During writing, the incoming number replaces the number previously held in the register.

C. Timing. Digit pulses, read pulses and write pulses appear if at all at times n.T. During either reading or writing an initiating pulse (a Read or a Write pulse) preceeds the first digit of the N-digit number by one clock period. (It is convenient to consider the initiating pulse to appear at time 0 and as a consequence, the least significant digit at time 1 and the most significant digit at time N.). The pulses required and the times at which they occur are shown in the table of fig. 6. External connections are made to the Pulse Generator by means of four bus bars. These carry the Read, Write, p1 and p2 pulses. The Generator contains a 6-stage binary counter which has provision to accept a clear-to-zero pulse. Connected to each stage of the counter is a Binary Gating Tube which at all times represents the state of the binary counter. These tubes form a set of coincidence gates and connected as shown, allow a pulse to pass when the counter registers the number 40, (binary number 101000).

In operation, either a Read or Write pulse opens a gate which admits ps pulses (shift pulses at times



A	auxiliary storage element.
S	digit storage element.
pa	pulses applied continuously at times $(n + 1/4).T$.
p1	"shift pulses" at times $(n + 1/2).T$.
p2	pulses at times $n.T$.

FIG. 5. N-STAGE SHIFT REGISTER.

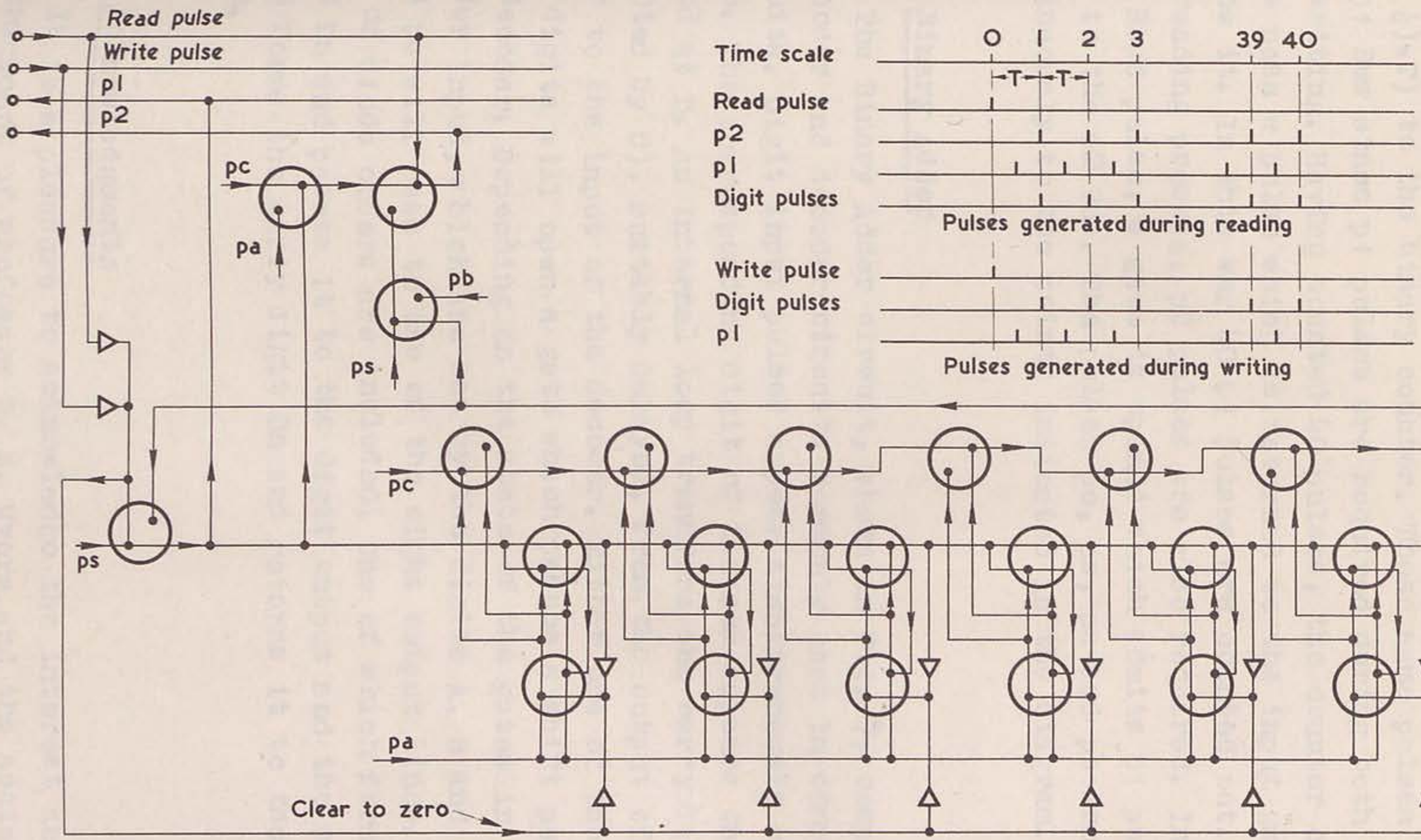


FIG. 6. PULSE GENERATOR FOR SHIFT REGISTER.

$(n + \frac{1}{2}).T)$ to the binary counter. These same pulses pass to the p1 Bus since p1 pulses are required during both reading and writing. Having counted 40 pulses, the counter coincidence gates pass a pulse which is returned to the input gate and closes it. In this way 40 p1 pulses are counted out. During the reading process, p2 pulses are also required. In response to a Read pulse, a gate is opened which admits p1 pulses to pass to the p2 Bus. The pulses pc, ps, pa and pb are applied continuously to the points indicated in the diagram.

Binary Adder

The Binary Adder circuit, shown in fig. 7, comprises the coder and decoder circuits commonly used in computing circuits. Digit input pulses appear simultaneously at A and B. The corresponding digit of the sum appears one clock period at D. An internal loop transfers the carry digit, (denoted by C), suitably delayed, from the output of the coder to the input of the decoder. Either one of the A, B, or C digits will open a gate which passes a shift pulse to the decoder. Depending on the state of the gates in the decoder input, which are set by the digits A, B and C, the pulse ps will pass to one of the eight output lines. Two sets of diode coders are included, one of which forms the sum digit Dn and passes it to the digit output and the other which forms the carry digit Cn and returns it to the decoder input.

4. Acknowledgments

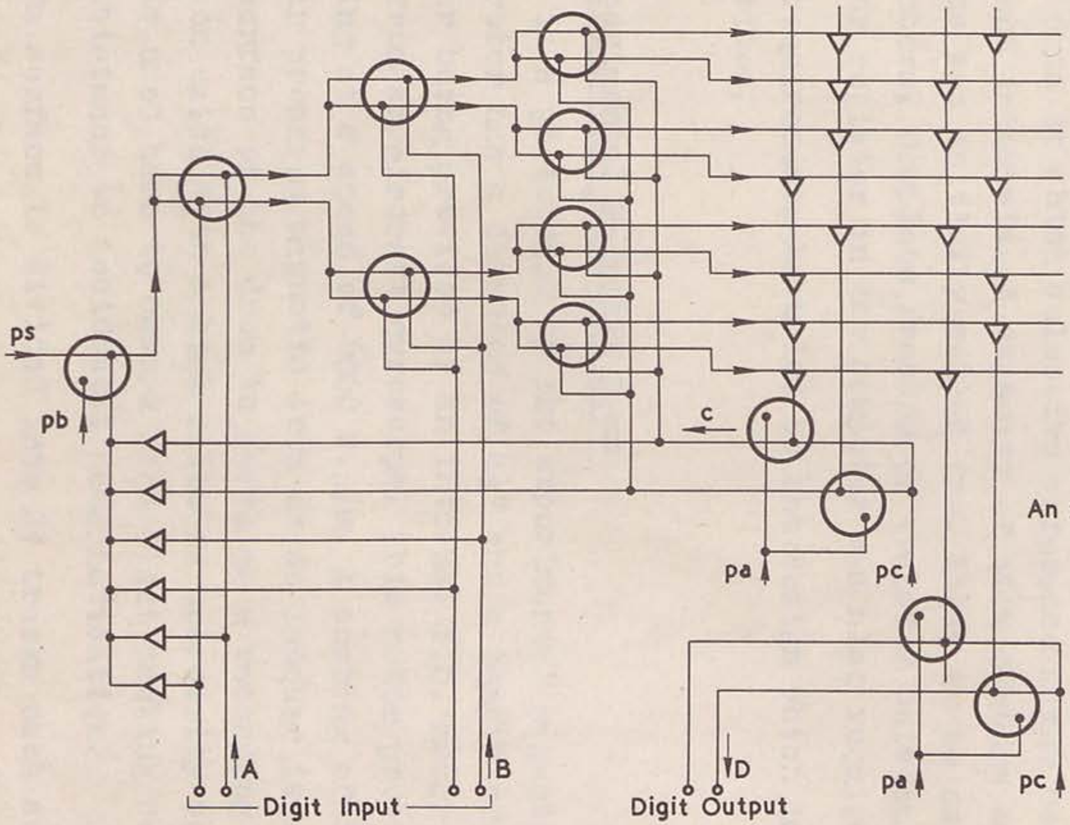
It is a pleasure to acknowledge the interest and encouragement of Professor D. M. Myers and the assistance of Mr. L. G. Bellamy (C.S.I.R.O.) for the manufacture and assembly of experimental tubes. The technical assistance and co-operation of Mr. D. Hollway (C.S.I.R.O.), Mr. R. E. Aitchison (University of Sydney) and the staff of the Valve Laboratory, is gratefully acknowledged.

B. F. Cooper: A Magnetic Drum Digital Storage System

1. Introduction

Storage of binary digital data on magnetic drums is now well established in modern electronic computing practice. Features which have led to the adoption of this form of store include: (1) compactness of the store, (2) non-volatility of the stored information, permitting indefinite retention of the latter even when the equipment is shut down, and (3) ability, notwithstanding the latter feature, to alter the information at will. Opposed to these three advantages is the disadvantage that mechanical motion is involved and access to the stored information is, therefore, slow in comparison with the all-electronic storage systems. However, the magnetic drum usually serves as an auxiliary store to which relatively infrequent reference is made, so that a small access time is not of great importance.

Integration of an auxiliary drum storage system into the structure of the main computer can be approached in two ways, depending on whether or not it is desired to store on the drum programme data which will ultimately be required at high speed. When such material is stored the system is organised in such a way that the progress of a computation can be temporarily halted while a large block of information is transferred from the drum to the high speed store.



An	Bn	Cn-1	Cn	Dn
1	1	1	1	1
1	1	0	1	0
1	0	1	1	0
1	0	0	0	1
0	1	1	1	0
0	1	0	0	1
0	0	1	0	1
0	0	0	0	0

$$An \times 2^m + Bn \times 2^m = Cn \times 2^{m+1} + Dn \times 2^m.$$

FIG. 7. BINARY ADDER.

This can be done during one drum revolution occupying say $\frac{1}{50}$ second, so that if the transfer operation is only required at intervals of several seconds, the lengthening of the computation on this account will be negligible. An arrangement of this type has been adopted with the Manchester machine (29) but it requires that the design of the high-speed and auxiliary stores should be carefully integrated ab initio.

In the case of the Radiophysics machine the drum store was designed as an adjunct to the main computer when the latter was almost complete, and it was required only for the storage of data to which selective reference might be made at infrequent intervals. A capacity of 1024 numbers each of 20 digits was initially called for, later to be extended to 4096 numbers. Complete freedom of transfer between the drum and any register in the computer was also required.

These requirements have led to the design which is now to be described.

2. The Experimental Magnetic Drum

Fig. 1 is a photograph of the experimental magnetic drum. The rotor has a diameter of $4\frac{1}{2}$" and a length of 3", motive power being provided by an internal D.C. motor which was taken from an aircraft gyroscope. This motor provides quiet running at a speed of 6000 r.p.m. A coating of a commercially prepared magnetic iron oxide lacquer is sprayed on to the surface of the drum to serve as a recording medium. Magnetic iron oxide with a high ratio of coercivity to remanence is used here to ensure high digit packing densities and high resistance to accidental demagnetisation.

The drum surface is divided into 21 tracks each served by a magnetising head which functions for both reading and recording. For reasons of mechanical convenience the heads are mounted in two staggered rows. Of the 21 tracks, 20 have numerical data recorded on them and on the 21st is a permanently recorded timing waveform which serves to index the recording positions on the drum.

A number of 20 digits is recorded as a row of magnetic "marks" spread transversely across the drum with one digit in each track. 1024 rows spaced peripherally at 75 per inch make up the full capacity of the drum.

This general scheme of "parallel" recording has been described previously by Booth (2) and by Hill (26).

Although the main computer operates in the serial mode, that is, numbers are transmitted as a serial train of pulses on a single circuit, the serial mode of operating the drum, i.e. with adjacent digits on a track grouped together to form numbers was not adopted for the following reasons. In the first place the computer timing relationships are based on a mercury delay line store with a digit pulsing rate of 333 Kc/s and a circulation time of 1 millisecond. As it was considered too difficult to match this speed in the drum system it was decided to operate the drum asynchronously with a buffer storage system interposed between the drum and the computer for the temporary holding of numbers being transferred. This being the case, the drum system could operate in whatever mode was best suited to its structure and it is easier to perform selection in one step from one of 1024 rows of numbers than to perform selection in two steps: (a) by track and (b) by position in the track.

3. Recording method

The manner of recording digits in a track is illustrated

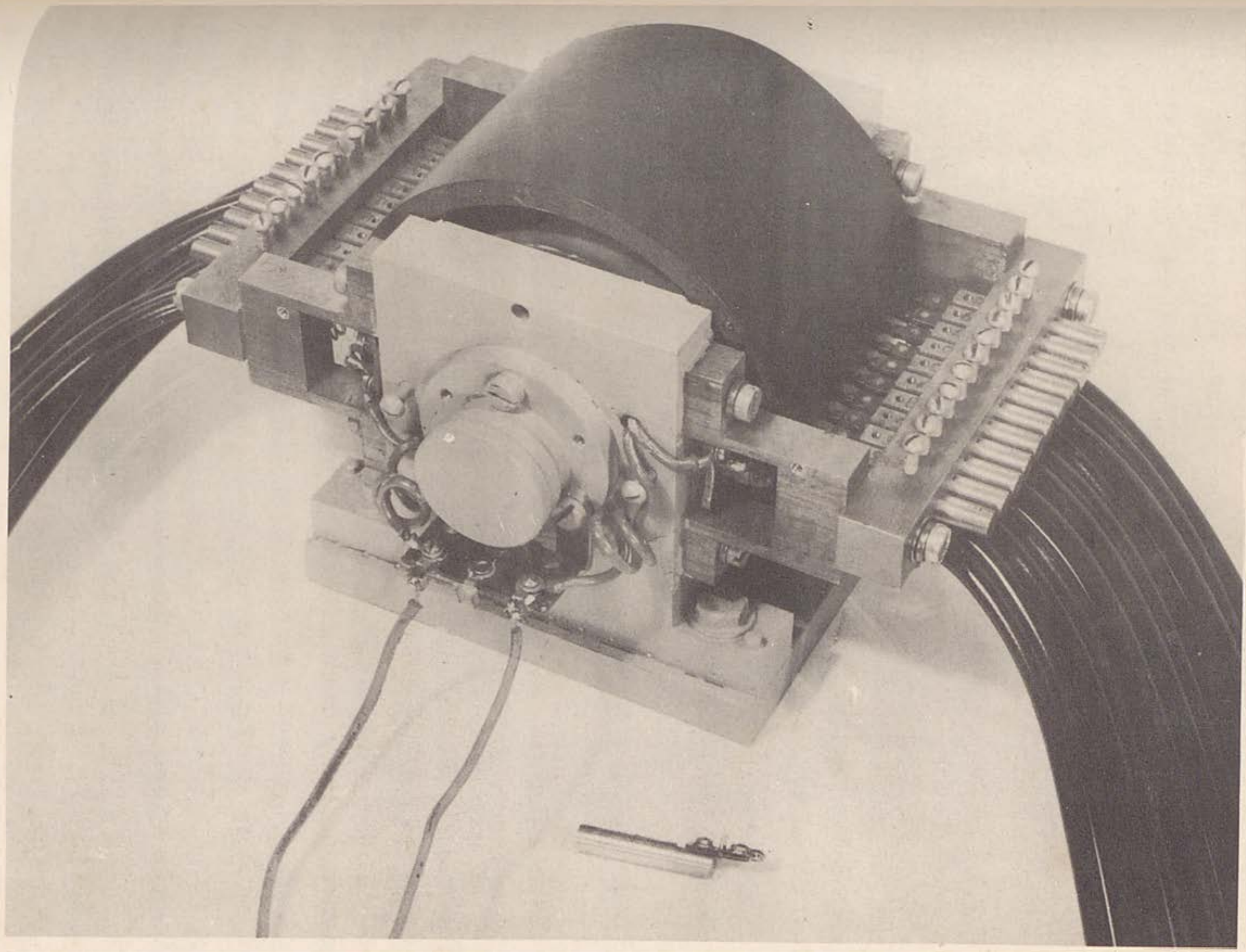


Fig.1 - View of experimental magnetic drum

in more detail in fig. 2, which is a diagrammatic cross-section of a track and its recording head. A recording head consists of a single strip of .003" mumetal suitably bent to shape and carrying a 100 turn magnetising winding of 36 B & S wire. Pole tips are separated by a .0015" non-magnetic spacer and a clearance of .0015" is maintained between the head and the drum. Operation in contact is not possible because of the high peripheral speed. Each head is provided with a thyratron pulser, as shown in fig. 3, capable of delivering a positive or negative current pulses for recording binary "0" or "1" digits. The duration of these pulses, being about 1 microsecond, is short enough to effectively "freeze" the motion of the drum, and the peak current employed, namely 1 ampere, is sufficient to produce local magnetic saturation in the recording medium.

Recording positions or "cells" on the tracks are defined by the intervals of the clock track and are identified by counting from a small gap in which nothing is recorded. The cells are ideally discrete but at the adopted spacing of 75 per inch they actually overlap somewhat. Initially the tracks are magnetised uniformly to the "0" level by passing D.C. of negative polarity through the heads while the drum is rotating. After the initial preparation a "1" may be recorded in any cell by the application of a positive current pulse to the head at the appropriate instant as shown in fig. 2a. This results in the residual magnetism being locally reversed, leaving a small hump in the flux pattern. Subsequent recording operations on this cell will have a negligible effect if a further "1" is recorded but if a "0" is recorded the flux pattern will be restored approximately to its initial state. Thus no special erasing operation is required.

To read the content of the cells the voltage induced in the head by the flux pattern is amplified and fed to suitable reading circuits. Fig. 2b shows the flux waveform induced in the head by the passage of cell numbers 1, 2, and 3, which have had digits 1, 0, and 1 recorded in them. The voltage waveform induced in the head is the time-derivative of the flux waveform so that a waveform of the shape indicated in fig. 2c appears at the amplifier output terminals. The amplifier output is applied to one input of a coincidence gate circuit, into the other input of which is fed a short position-selector pulse, derived in a manner which will be described later, and phased relative to the amplifier output as shown in fig. 2d. If the position selector pulse coincides with a positive peak in the amplifier output, resulting from the previous recording of a "1", an output is derived from the coincidence gate which sets an output "flip-flop" to the state "1". If the cell contains a "0" the small residual ripple in the output waveform cannot actuate the coincidence gate. An oscillographic record of the amplifier output waveforms resulting from various digit configurations is shown in fig. 4. The peak to peak voltage delivered to the amplifier by the head is 5 millivolts.

4. Auxiliary electronic equipment

A block diagram of the equipment necessary to operate the magnetic drum in conjunction with the computer is shown in fig. 5. Contact with the computer is established through a "drum sequence unit" which controls the cycle of operations whenever a drum transfer is called for.

In the case of a transfer from computer to drum, a "drum destination" control line is made active by the computer controls and a 10 digit binary number specifying

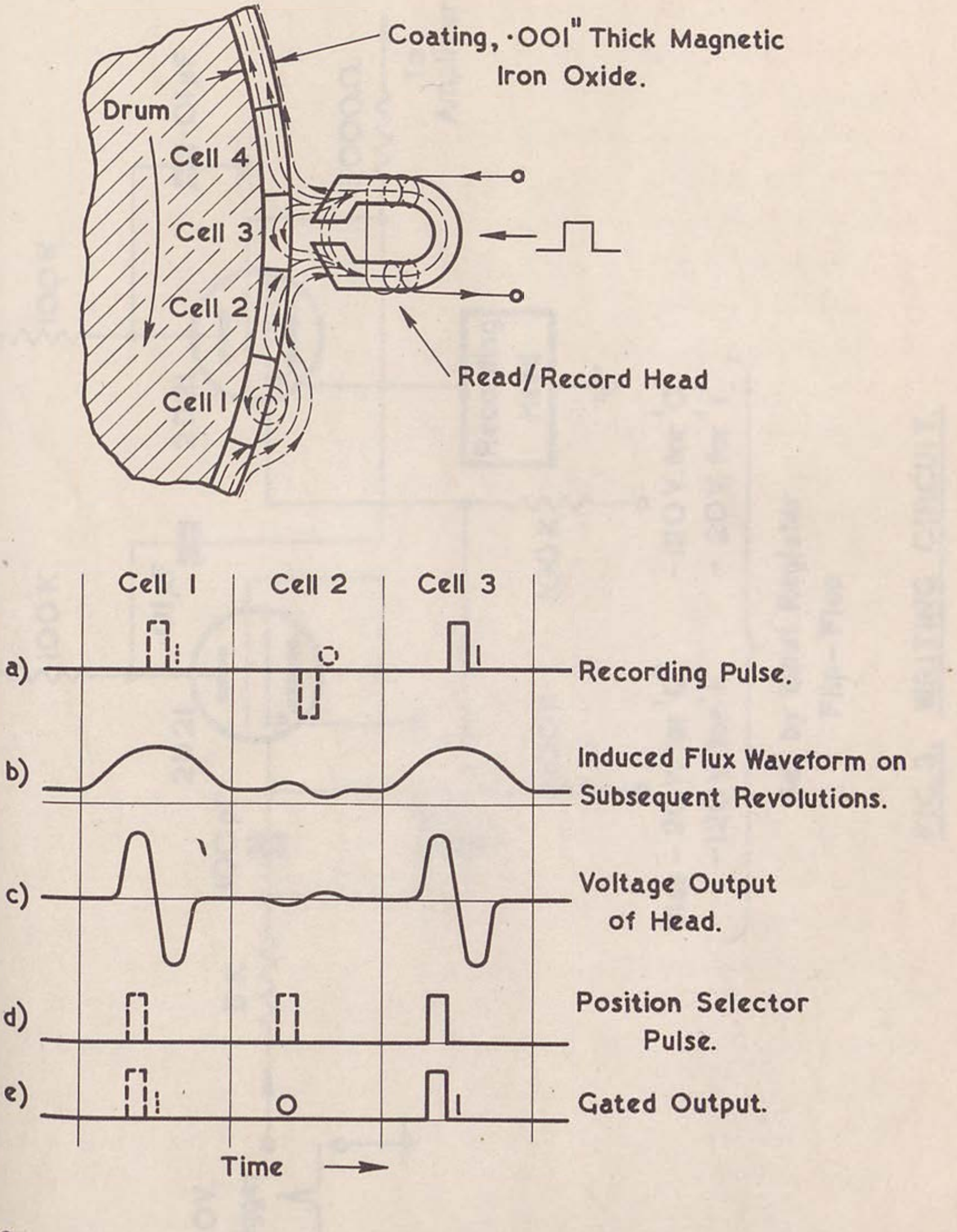


FIG. 2. DETAILS OF PICK-UP HEAD & FLUX & VOLTAGE WAVEFORMS.

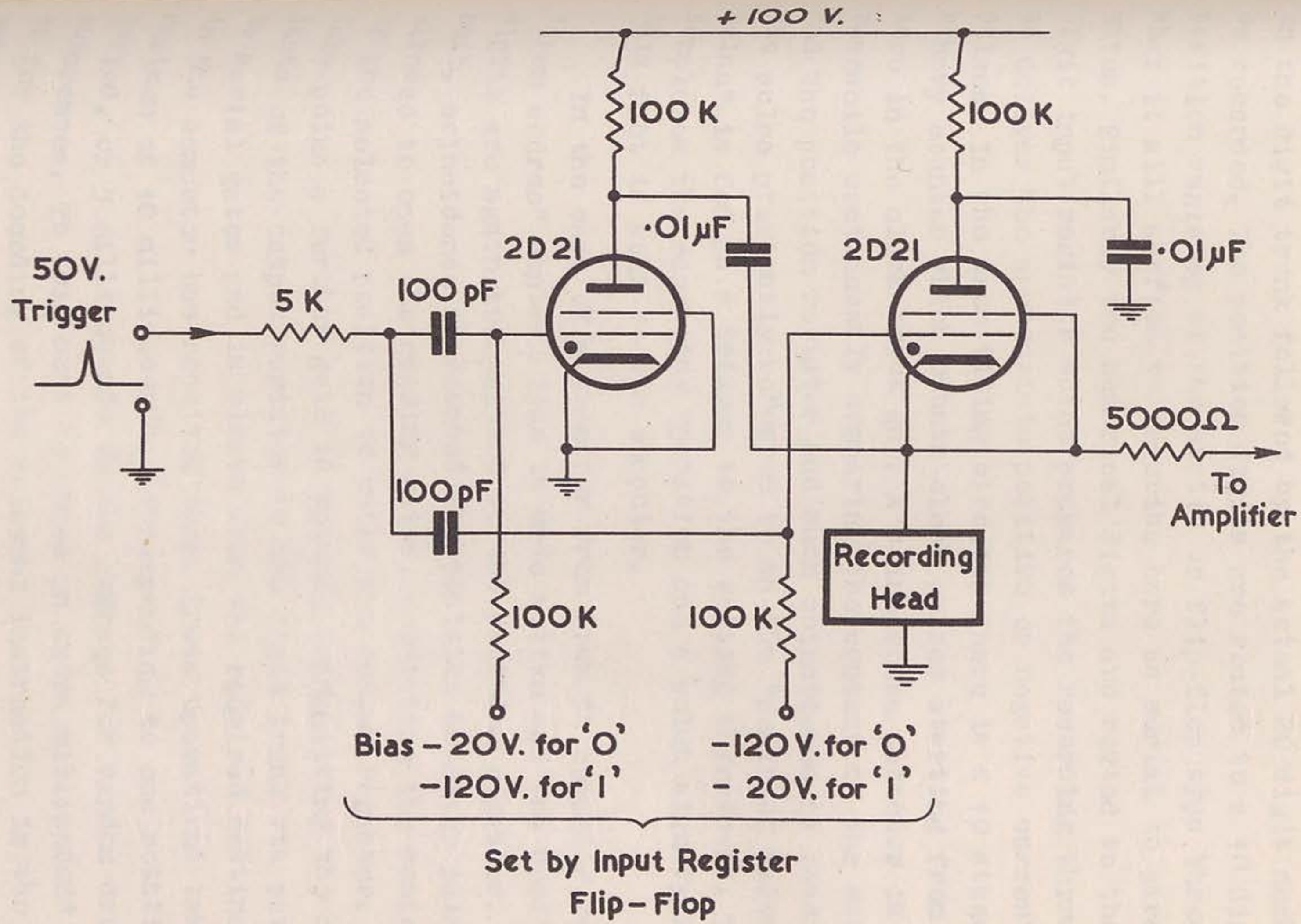


FIG. 3. WRITING CIRCUIT.

which one of the 1024 recording positions is required appears on the digit trunk followed by the actual 20 digit number to be recorded. The position digits are routed to a 10 digit position register of the static or flip-flop type through what it will suffice to describe here as serial to parallel gates. Similarly the numerical digits are routed to the 20 digit input register which prepares the recording thyratrons to deliver the appropriate positive or negative current pulses. In the drum timing circuits there is a 10 stage binary counter which counts clock pulses starting from zero in the clock track gap. A coincidence detector is meanwhile continuously comparing the content of the counter and the position register and when coincidence is reached the pulse previously referred to as the "position selector pulse" is fed as a trigger to the writing thyratrons. This completes the recording operation and a pulse signifying this fact is sent to the computer.

In the case of a transfer from drum to computer, a "drum source" control line is made active and the position digits are again transmitted to the position register. When, next, coincidence is reached the position selector pulse is allowed to open the reading gates, permitting the contents of the selected position to enter the output register. At this point a further gate is opened, transmitting the contents of the output register to the digit trunk via parallel to serial gates and is closed when the required destination in the computer has received them. These operations take a maximum of 10 milliseconds, corresponding to one rotation period, or 5 milliseconds on the average for random drum references. To this must be added an extra millisecond or so for the decoding of the relevant instruction in the computer.

A total of some 280 valves is used in the above circuits, and this number could be cut down to 200 by eliminating the input, output and position registers and sharing certain registers in the main computer. However, the provision of separate registers has led to flexibility in testing the experimental system. It is estimated that expansion of the drum capacity to 4096 numbers will require the addition of only 60 - 70 valves.

5. Reliability test

Little experience has been gained so far in actual computing operations with the drum but as a test of its reliability it has been run for several weeks on a closed cycle of operations. In this test the sequence unit is connected to perform writing and reading operations on alternate revolutions, i.e. at 50 cycles per second and the position register is connected as a counter with units being added to its contents at the beginning of each writing cycle. After a reading operation the contents of the output register are immediately transmitted into the input register with a left shift of one digit position.

Initially a number 00000,00000,11111,11111 is set into the input register and the cycle is started with this number being transmitted to position "0". As the cycle progresses the number 00000,00001,11111,11110 should appear in position 1, 00000,00011,11111,11100 in position 2, and so on, "End around carry" from the 20th to the 1st digit position being permitted, so that the spiralling effect can continue indefinitely. After 1024 cycles requiring approximately 20 seconds the progression of numbers should return to position 0 with the number 00000,01111,11111,10000 being inserted there, provided no errors have occurred in previous reading and writing operations. After 5720 cycles the number

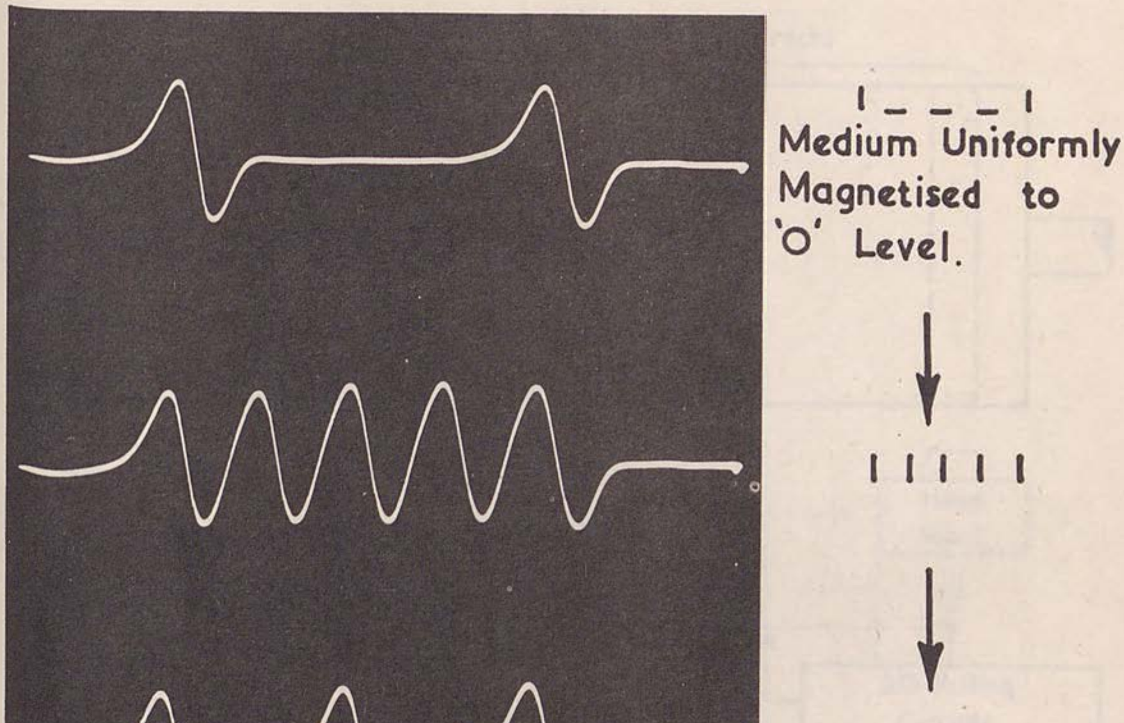
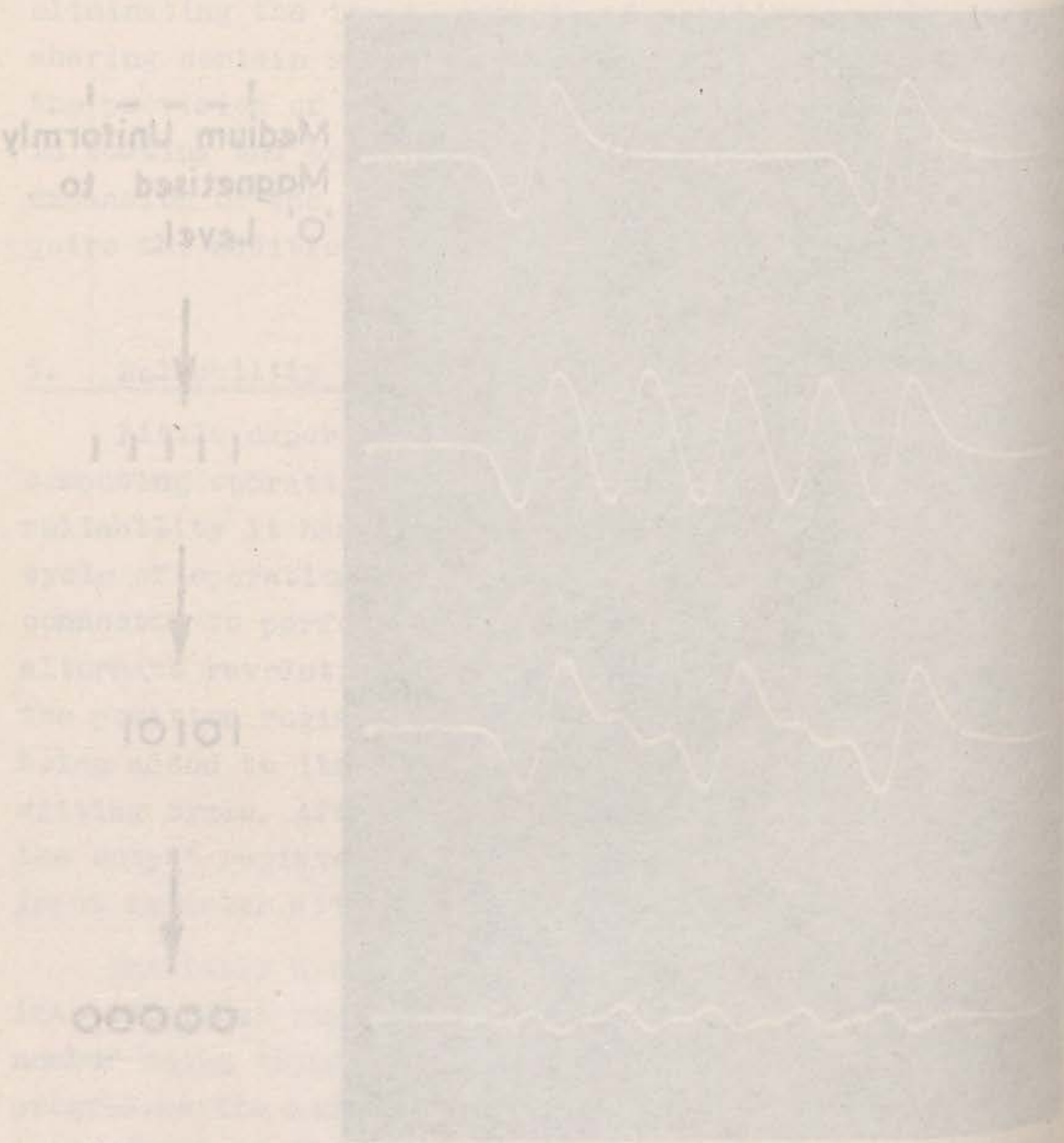


Fig.4 - Oscillographic record of output from magnetic drum



Read Out Waveforms.
Magnetic Drum, 6000 R.P.M., 75 Digits per Inch.

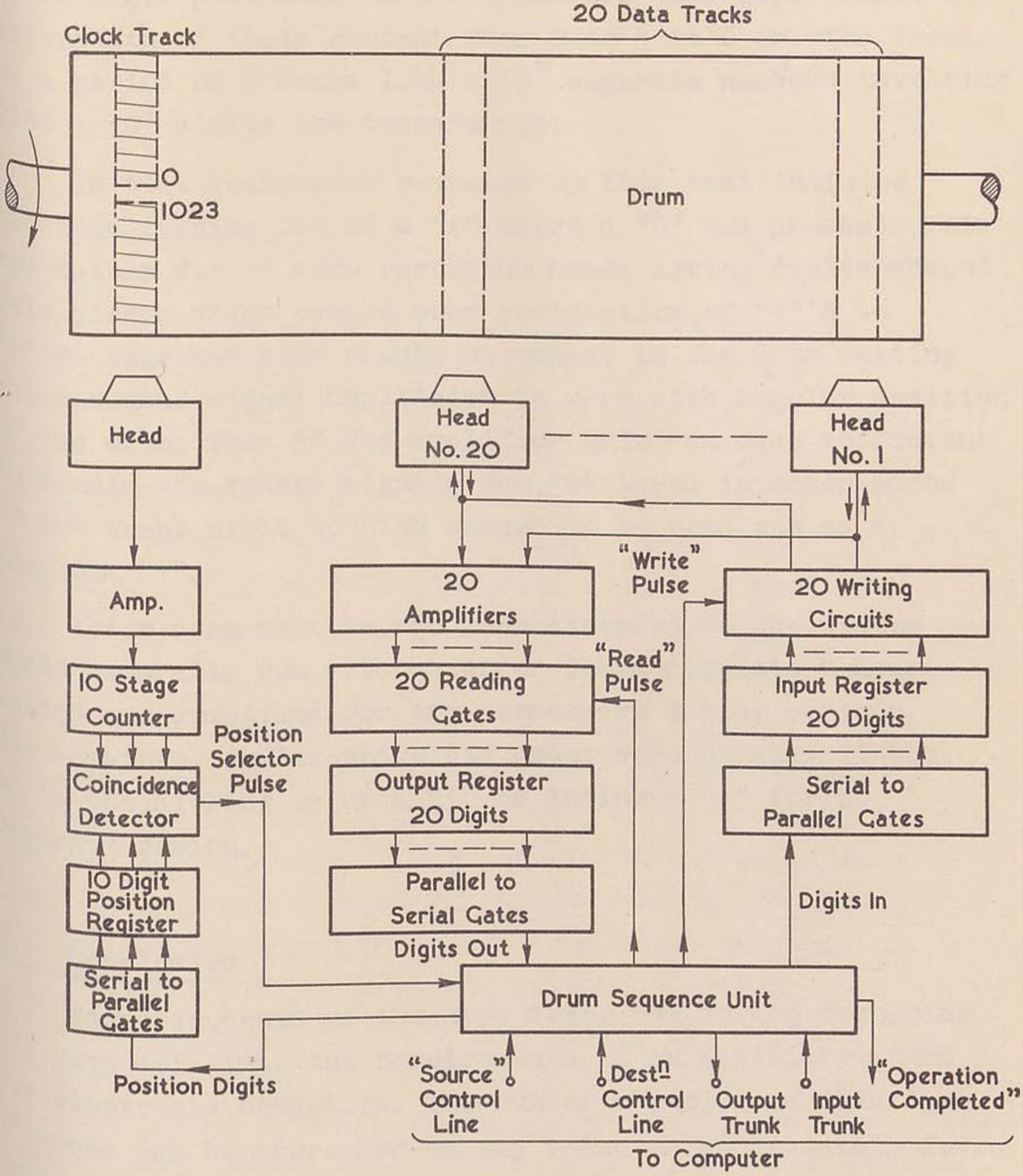


FIG. 5. AUXILIARY ELECTRONIC EQUIPMENT.

00000,00000,11111,11111 should reappear in position 0 and all 20480 digit positions on the drum will have experienced an alteration of their content from 0 to 1 to 0 or vice versa. In a period of 8 hours 1.44×10^6 separate numbers involving 2.88×10^7 digits are transferred.

Initial weaknesses revealed by this test included spurious reading out of a "1" where a "0" was present. This was mainly due to some recording heads having faulty shaped pole pieces which caused poor restoration of "1"'s to "0"'s, together with slight uneveness in the drum coating which caused signal amplitudes to vary with angular position of the drum. Thus if the amplifier gain was made sufficient to handle the weaker signals the "0" level in other parts of the track might be high enough to be read out as a spurious "1".

After such matters had been attended to the system would generally run without error for a complete 8 hour period and sometimes for two successive 8 hour periods. The remaining faults which did occur were equally liable to be the picking up of a "1" or losing a "1" for no apparent reason.

6. Conclusion

Magnetic drums of the type described in the foregoing fit very well into the requirements of an auxiliary store for electronic computers. The number capacity of this type of store can be increased to any value which is likely to be required with little increase in engineering complexity. Furthermore, since considerable time is consumed in loading a store of high capacity the feature of retention of the stored information during a shut down should prove to be a considerable advantage.

Discussion

B. F. C. COOPER - replying to questions, explained that the transfer of information from the magnetic drum to the main store in the C.S.I.R.O. machine is not carried out systematically in blocks but instead, individual numbers are read from the auxiliary store. As the drum is used principally for storing function tables, this does not reduce the overall speed of the machine seriously. Mr. Cooper asked Professor Hartree if he would explain in some detail the functions of the auxiliary store in the Manchester machine and also the method of programming transfers between main and auxiliary store.

D. R. HARTREE - said that he was not familiar in detail with the Manchester machine but commented generally on magnetic storage. In the Harvard Mk.III machine the magnetic drum is used as a main store. It uses a number of comparatively large magnetic drums which are mechanically coupled, and each track has 2 to 4 reading/writing heads to reduce access time. The EDSAC, on the other hand, has no subsiduary store. Commenting on special tubes, he said that a binary adding tube is about to be produced commercially in Canada. However, before any special tubes are acceptable for computers they must be of proved reliability and also in ready supply.

D. M. MYERS - said that the binary gating tube described by Mr. Speedy could be a practical production job. The adder being produced overseas is rather more complex.

K. S. BROWN - stated that the Canadian tube is now available commercially. It is called the Additron. He paid a tribute to the Hollway tube, pointing out that it is the first high vacuum decimal counter tube to be produced. It is most ingenious and further development would be sure to improve its upper frequency limit. Gas-filled counter tubes are commercially available but their upper counting limit is of the order of 20 to 30 Kc/s.

C. B. SPEEDY - replying to a question on the use of crystal triodes in computer work said that they appeared to offer promising possibilities. He knew of no overseas machines that were as yet employing them, although crystal diodes were being used in great numbers. No crystal triodes were being used in Australia because of shortage of supply.

G. E. BARLOW - asked whether anything had been done locally to synchronise the pulses from a magnetic drum with the master clock pulses of the C.S.I.R.O. machine.

B. F. C. COOPER - said that synchronisation had not been attempted. The master pulses have a frequency of 330 Kc/s and the pulse repetition rate of the magnetic drum is of the order of 100 Kc/s, so that direct synchronisation is not possible. If it were, it would require rather a complex servomechanism. In the C.S.I.R.O. machine, transfers between drum and the main store were carried out in two stages, via a static register.

W. R. BLUNDEN - indicated that work had been done on this problem in the C.S.I.R.O. Mathematical Instruments Section where an electrostatic storage tube is being built. The pulse rate for the electrostatic store is about 120 Kc/s and a 7-inch drum, rotating at 6000 r.p.m., storing about 55 digits per inch, gave a similar pulse rate. Direct synchronisation is being attempted. The drum is driven by a velodyne which, in addition to its main first derivative feedback circuit, employs a special synchronising circuit which produces a signal proportional to the phase difference between drum pulses and the reference pulses. At present the results were not very promising, mainly because the velodyne being used was rather too small for the job.

D. L. HOLLWAY - replying to another question from Mr. Barlow said that the input impedance of the counter tube is 10$\mu\mu$F.

XI. Some Analogue Computing Devices

by D. M. Myers

1. The Role of the Analogue Machine

During the last decade, the relation between the use of digital and analogue machines has undergone a very pronounced change in favour of the former, as a result of the developments in automatic digital computing which have been described in the early discussions at this Conference. It must now be conceded that, with few exceptions, a numerical solution of a mathematical problem can be carried out more quickly and more accurately by a digital machine than by an analogue machine. Furthermore, a digital machine can be applied to the solution of any problem which can be translated into a series of simple arithmetical and logical operations, and this is possible in the case of most scientific problems of practical significance. The modern digital machine is a general purpose machine which, with its speed and accuracy, is becoming a most valuable tool in the hands of research and industry.

What, then, is the role of the analogue machine? Under what conditions is it preferable to the digital machine, and what is its future, in the face of the increasing power of the latter?

To find an answer to these questions, we must look beyond the mathematical problem itself, and consider its physical context. As a simple example, let us consider an analogue machine which is well-known to all of us, and which performs continuously a rather intricate set of mathematical operations. I refer to the electricity meter, which determines how much we must pay for heating and lighting our homes.

Superficially, it simply measures the amount of energy - the number of kilowatt-hours consumed. In order to do this, it accepts data in the form of three continuously variable quantities: voltage, current and time; it multiplies the first two of these together and integrates the product with respect to the last. And it does all of this for an extremely low cost of production. It is most unlikely that the electricity meter in its present form will be superseded in the foreseeable future by a digital machine. This meter is digital only in the presentation of the results of its computations, and fundamentally it is a machine of the analogue class.

This is perhaps an extreme case, but it illustrates a significant factor. Multiplication and integration can be done easily and rapidly by digital methods; the reason for the choice of an analogue device lies in the context, including the form of presentation of the data. These data - voltage, current and time - are constantly varying quantities, which appear in a form suitable for immediate digestion by an analogue machine. Their conversion to digital form would in itself be a considerable problem.

A more advanced illustration of the same point occurs in the case of railway schedule computations, for which a form of differential analyser has been specially designed and built in the U.S.A. The problem occurs frequently in a country with a rapidly developing railway network, and there are obvious advantages in using a machine which requires no special setting-up each time the problem occurs.

Another consideration of similar importance is the formulation of the problem itself. Many problems, particularly in engineering practice, are readily susceptible to treatment with the aid of a convenient analogy. To solve them by digital methods requires their re-statement in mathematical symbology, so that they can be broken down into arithmetical routines. This is sometimes impracticable.

In general, the position may be reasonably stated as follows:-

(1) The digital machine has the overall advantage in generality, speed and accuracy.

(2) The analogue machine is designed for a limited purpose, but in serving that purpose, it frequently has the advantage owing to the context of the problem and the form of presentation of incoming data or of its output.

As a corollary, the scope of the analogue machine is usually greatest where either the data or results appear in the form of continuously varying physical quantities or where the problem involves relationships which cannot readily be converted to symbolic form.

This point should be stressed because it suggests a rather different way of examining the relative merits of analogue and digital machines. Machines may be classified according to the types of mathematical problem to which they can be applied:- e.g. harmonic analysis, differential equations, general purpose, etc. On the other hand, they may be subdivided according to their "user" classification: e.g. crystal structure analysis, determination of railway schedules, electron path plotting, etc. In my earlier remarks, I have suggested that the role of the analogue machine lies mainly in solving problems in the latter classification and it is appropriate at this Conference that a brief account should be given of some of the more important of these; the former classification is becoming more and more the province of the digital machine.

2. Machines for Special Applications

A very widespread type of analogue machine designed for a specific practical use is the anti-aircraft predictor, or more generally, the whole range of instruments used for determining continuously, from observations of a target, the necessary data for the projection of missiles to destroy it. The problem is a continuous one in real time and cannot be treated with detachment as a mathematical exercise to be solved in the laboratory when convenient. Each solution is required within a very small fraction of a second after acceptance of the data on which it is based, and in a form suitable for immediate use. The analogue machine meets the requirements very well, and has been almost unopposed in this field up to the present. There are obvious difficulties in the use of digital machines for this purpose, but they do not appear to be insuperable, and it is reasonable to predict that analogue machines will be replaced, at least in part, by high-speed digital machines for gunnery control in the future.

It is reasonable to include also the electrolytic tank * among those devices used to satisfy a "user" requirement. In general, its usual purpose is to solve the Laplace equation for given boundary conditions, and to this extent it may be regarded as applicable to a specific kind of mathematical problem. However, a great deal of ingenuity has been applied to the method of presentation of the solution in such a way as to satisfy particular "user" requirements. One example of this is the determination of the path of an electron when passing rhough a system of electrodes such as exists, for example, in an electron

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* For fuller details, see ref. 33a, pp. 112 et. seq.

microscope or a linear accelerator. If the electrode system is represented to a suitable scale in the electrolyte, the potential of any point in the field may be determined by measurement and analogy. Equipotentials can then be plotted, and from them the electric intensity determined at any point. A knowledge of both the potential and intensity enable the path of an electron to be found. Various devices have been used to avoid the need for the plotting and computing, and one such device, due to Mr. D. L. Hollway, is available for demonstration during the conference. This instrument, attached to a pantagraph which locates the probe in an electrolytic tank, enables an operator to trace an electron path directly, without auxiliary plotting or computing, by steering the instrument across a drawing board in such a way as to maintain a null reading on an indicating instrument.

In the cases just described, the problem can usually be stated in reasonably simple mathematical form; the chief interest in the machines lies in the methods of accepting the data and presenting the result. The analogue machine has a special application also to those problems which cannot be conveniently stated symbolically, either because of their complication or, for example, because of the nature of their boundary conditions. Many of these problems are solved in practice by direct analogy, using convenient scale factors, such as in the aerodynamic wind tunnel and the towing tank. An example of great practical importance is the network analyser *, which is used for the study of interconnected electrical generating systems. This instrument solves a set of simultaneous equations, usually in complex variables, but the statement of the problem in such a form would usually be a task of considerable magnitude; this is simplified by

* See, for example, refs. 23, 51.

the use of the network analyser. Most instruments of this type depend on a scale model of the system, sometimes operated at a comparatively high frequency to reduce the physical size of the components. The analogy used may be (1) an indirect one, as in the case of the Blackburn analyser in which transformers are used to replace the normal circuit elements.

Many analogue machines built for particular practical requirements embody components which may themselves be classed as analogue machines of more general application to mathematical problems. It is desirable at this stage to outline very briefly the way in which some of the simpler mathematical functions may be represented in analogue form.

3. The Simple Arithmetical Functions

Analogue devices for performing the simple arithmetic processes, such as addition or division, are in such common use that their mathematical significance is frequently overlooked. The differential of a motor car constrains the rotations of the two rear wheels in such a way that their sum is proportional to the rotation of the driving shaft, without imposing any constraint on the relative rotation of the two wheels. Thus it is basically a device for adding (or subtracting) two quantities expressed as rotations.

In the automatic totalisator (28), familiar to race-goers, this principle is used, together with a number of dividing mechanisms based on the properties of similar triangles, to carry out a rather more complicated problem, consisting of repeated additions and divisions.

The slide-rule and the planimeter are too well known to require description. The planimeter and the integraph have been extended by various workers to evaluate not only definite and indefinite integrals but also first, second and higher moments, and even to integrate differential equations within a limited field.

All the above operations can be carried out, either directly or by finite difference methods, by digital machines; the choice of method is usually based on convenience.

4. Simultaneous equations

There is considerable practical interest in the mechanical solution of simultaneous algebraic equations because of their frequent occurrence and of the tedious nature of their solution by conventional methods if the equations are in more than four or five variables. Several analogue devices have been built for this purpose, the best known being that due to Mallock (33) at the University of Cambridge. This instrument depends on Faraday's law of electromagnetic induction and consists of a number of transformers, each having a number of tapped windings. The equation is represented by a suitable interconnection of the windings and the number of turns in each winding is set according to the appropriate parameters in the equation.

A mechanical device, due to Wilbur (54) in the U.S.A., consists of a set of tipping plates, mutually constrained by a system of pulleys and steel tapes.

In machines of this kind, the physical size depends roughly on the square of the number of variables, so that in practice there is a limit of nine or ten variables. This difficulty may be partly overcome by using an iterative method of solution in a device which can deal with one equation only at a time. However, the solution still becomes very tedious if there are many variables.

It seems at present that digital methods are to be preferred for the solution of simultaneous equations; however, the principles of the machines described are of interest and may well be included in analogue devices for special applications.

5. Fourier Analysis and Synthesis; Solution of Polynomials

The synthesis of a periodic function from a series of harmonics, and the analysis of such a function into its harmonics, require very similar techniques. In an analogue solution, the first requirement is the generation of all the harmonics involved. A Fourier analysis requires also a product integration. These functions can readily be carried out by either electrical, mechanical or other methods, and a great deal of ingenuity has been applied to the use of various analogies *. The problem of analysis occurs very frequently in the practice of electrical engineering, the function to be analysed being presented in the form of a variable voltage. An analysis by electrical methods is very conveniently used, and several types of electrical harmonic analyser are available on the market.

When the data are photographic records or in graphical form, mechanical methods of analysis are often more convenient, and a number of such methods have been described.

In the solution of polynomial equations, it is a common practice to evaluate the polynomial expression for specific values of the modulus of the variable, and this can readily

* For example, see refs. 6, 15, 16, 25, 37, 41, 45, 50

by done by harmonic synthesis. Thus, a machine capable of performing harmonic synthesis can be used in the solution of the polynomial. Such a device, in mechanical form, is the Isograph (8, 36).

A special requirement for Fourier analysis arises from recent work in crystal structure analysis by diffraction methods. To those engaged in this work, the time and tedium involved in the interpretation of diffraction patterns needs no emphasis. The problem is one of Fourier analysis in three variables, and its solution involves the examination of trial molecules and the comparison of the results with experimental observations of the molecule under examination. The time involved in the computations relating to a single molecule, using normal facilities, is such as to impede very seriously the use of the results of diffraction experiments. The computations themselves could undoubtedly be carried out very quickly if a high-speed digital computer were available, but the presentation of the results in the form of a series of contour maps is itself an undertaking of some magnitude.

To overcome these difficulties, a device known as X-RAC (38) has been developed during the last few years by Professor R. Pepinsky at Pennsylvania State College in the U.S.A. It is a large and expensive machine, but it can present within several hours, on a cathode-ray screen, a series of contour maps relating to a crystal structure analysis, which would take many months to obtain with existing methods.

It is possible, of course, that future developments will make it practicable to carry out such a computation on a digital machine, and this possibility must not be overlooked. Nevertheless, the desired presentation is in "analogue" form, and there will obviously be a considerable demand for the use of instruments such as the X-RAC for a long time to come.

6. Differential and Integral Equations

The widespread occurrence of differential equations in scientific and industrial problems of all kinds gives the differential analyser a special place amongst analogue machines. As this subject has already been dealt with at some length during the conference, there is no need at this stage for more than a passing mention. The types of differential analyser already described depend on a continuously variable mechanical gear to carry out the process of integration. Several instruments have been designed to use other methods of integration, and electrical devices have had considerable application for this purpose. The relation between the charge on a condenser and the current flowing into it provides a convenient basis for integration. The principle of the electricity meter has also been used, as also has the velodyne, an electric motor which rotates at a speed proportional to an e.m.f. applied to it. These latter devices allow integration only with respect to real time, which seriously limits the flexibility of a differential analyser based on any of them.

Brief mention should also be made of the solution of integral equations, for which comparatively little work has been done towards the development of analogue devices. A machine known as the Cinema Integraph (24) was produced at the Massachusetts Institute of Technology, using an optical analogy, and appears to be the only important contribution in this field.

7. Statistical Analogues

In conclusion, I should like to draw attention to the possible use of analogies between statistical and continuous functions as a method of evaluating the latter. There is, for example, the statistically derived equation of the "random walk", which is analogous to the diffusion equation which occurs in many physical contexts. The use of this property has been put forward as a means of deriving solutions of the diffusion equation by digital methods. It is possible that a simple analogue device could be developed for this purpose, and it is suggested that the examination of statistical analogies may provide new bases of calculation by either digital or analogue machines.

It has been possible to give only a brief resume of the principles and methods involved in computing by devices of the analogue class, and many such devices have had to be omitted. However, there is an extensive literature on the subject, and the intention to-day has simply been to present the facts in perspective against the background of the most recent developments.

XII. Digital-Analogue Conversions

by W. R. Blunden

Introduction

Much has been said during this conference of the scope and application of digital and analogue machines. Developments in both fields have been spectacular so much so that the analogue computer and the digital computer are now accepted tools for mathematicians, mathematical physicists and engineers. Although the fields of usefulness of both these basic types separate out rather naturally, there appears to be a very promising new field for machines combining certain features of each. In order to effect this combination, it is essential to examine the possibilities of conversion between the numeral scales which are the working medium of the digital machine and the continuously variable physical quantity by which variables in the analogue machine are represented.

Before proceeding to discuss methods of digital-analogue conversion, i.e. conversion of digital information into analogue form and vice versa, a little more should be said of the "promising new field" mentioned above.

2. Field of Application of Digital Analogue Machine

The application of the digital and analogue machine is not based so much on the particular way in which these separate machines work but rather on the nature of the problem to be solved by them and in particular upon:-

(a) the accuracy required

(b) speed of solution

(c) the form of the input data

(d) the manner in which the answer is to be presented

(e) whether or not the solution is to be carried out in "real" time

This latter point is very important for if a problem involves real time as the independent variable, it becomes part of an actual physical process and the computing machine becomes one element of a more comprehensive control system. For example, the stabilizing of a position control servo-mechanism often demands that the mathematical operations of differentiation, integration and weighting and averaging with respect to real time be carried out on the error signal.

Care should be taken to appreciate the significance of real time in relation to speed. One generally assumes that digital machines work very rapidly and analogue machines slowly. However, in real time problems analogue devices often provide instantaneous solutions. In a digital device there must be necessarily some time delay between accepting the data and providing the results. This time delay may be made very small and a digital device will perform the computations to any prescribed accuracy standard and there thus appears to be scope for conversion of physical data direct to digital form and then carrying out the computation numerically.

As well as real time problems there are other problems requiring solution by numerical means in which the input data is most readily available in the form of curves and graphs and in which the answer is required in the same form. Problems of this type are common in engineering practice.

Another application is the employment of a digital type store for storing function data for an electronic analogue differential analyser.

In addition to computing there are possibilities for the employment of the high speed, large capacity digital store in control and transmission problems generally and means must therefore be provided for converting the stored digital data into its analogue equivalent.

3. Principles of Digital Analogue Conversions

Much work has already been carried out on techniques for effecting digital-analogue conversions especially in relation to pulse code modulation equipment (13,35) and also for coders and decoders. Although it is usually necessary to affect conversion both ways, the digital analogue conversion is the direct process whilst conversion from the analogue to the digital form is indirect and usually involves some form of discriminator. The principal of direct conversions will therefore be considered first.

In many cases the analogue data is generated mechanically and although mechanical methods may be used throughout the conversion processes it is usual to use some form of transducer to change the mechanical motion into electrical form and a servomechanism for the reverse process of changing the electrical signal back to mechanical motion. The digital analogue conversions are then performed electrically.

The digital quantity is stored in some form of static register such as a binary decade counter or shift register. It is well known that the wave-forms from successive stages of a binary decade if added in the ratio of ascending powers of 2, will produce a staircase wave-form with a rise equal to:

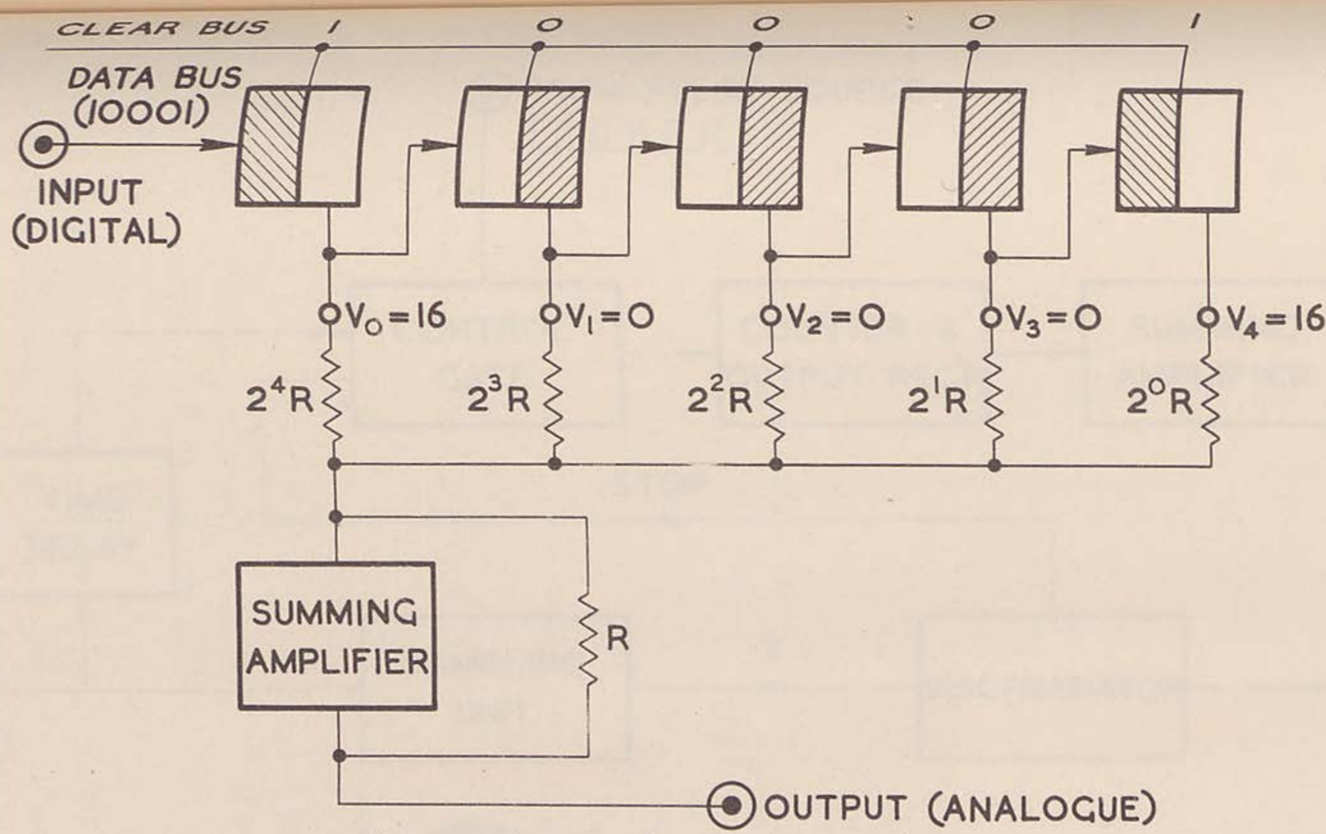
$$\frac{K.V}{2^{n-1}}$$

where V is the voltage excursion of the plates of the counter stage, n is the number of digits in the number to be converted and K is a scale factor.

For example, we may wish to convert five digit binary numbers. V could therefore be 16 volts and with K equal to unity the staircase wave-form would rise in steps of 1 volt to a maximum of 2V - 1, i.e. 31 volts. If the counter were set to represent 10001 a voltage of 17 would appear at the output of the summing amplifier. Fig. 1 shows a schematic diagram of a digital analogue converter for five digit binary numbers. The precision of the conversion depends on n and it is apparent that the accuracy of the conversion process depends directly on V. Stabilized supply voltages, limiting and clamping circuits may be employed to establish V to a high order of accuracy.

In addition to the storage register and the summing amplifier, a control circuit is necessary. Its primary functions are to control the entry of the digital data into the register and ensure that it is cleared prior to new data being fed in.

The inverse process of converting analogue quantities into a numerical code may be carried out with the equipment described above together with a discriminator and additions to the control circuit. The analogue quantity is presented to one side of the discriminator and the output from the summing amplifier to the other. The control signal initiating



$$V_o = \left(\frac{1}{2^4}V_0 + \frac{1}{2^3}V_1 + \frac{1}{2^2}V_2 + \frac{1}{2^1}V_3 + \frac{1}{2^0}V_4\right)$$

$$= \left(\frac{16}{2^4} + \frac{0}{2^3} + \frac{0}{2^2} + \frac{0}{2^1} + \frac{16}{1}\right)$$

$$= 17.$$

FIG. I. BLOCK DIAGRAM OF DIGITAL TO ANALOGUE CONVERTER.

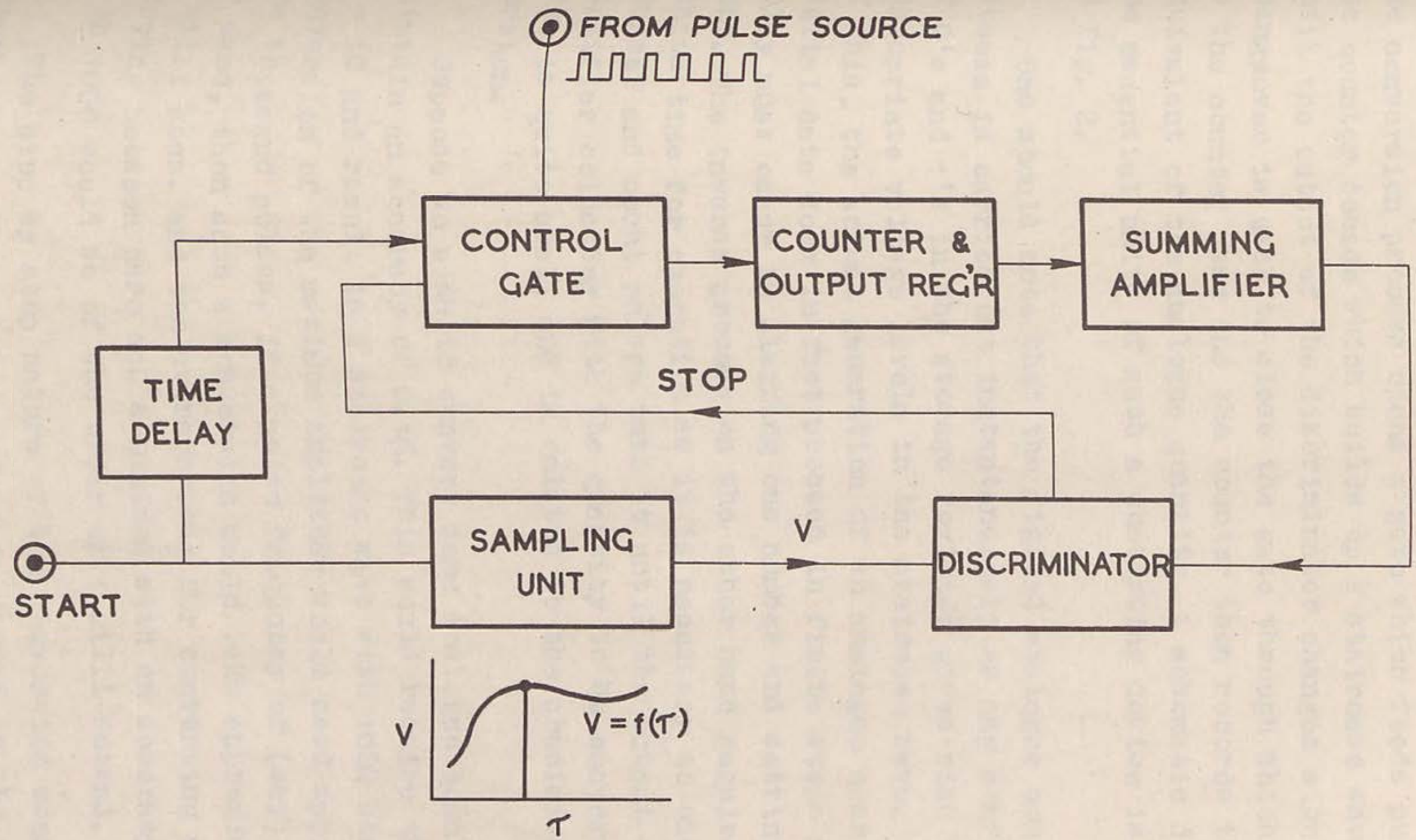


FIG. 2. BLOCK DIAGRAM OF ANALOGUE-TO-DIGITAL CONVERTER.

the conversion process opens a gate which feeds pulses to the counter decade which builds up a staircase wave-form until the output of the discriminator changes sign. This changeover is used to close the gate through which pulses to the counter pass and the counter then records the digital equivalent of the analogue quantity. A schematic diagram of the essential units of such a converting device is shown in fig. 2.

One should note that the digital analogue conversion process is carried out instantaneously as any configuration of 0's and 1's in the storage register gives rise to the appropriate voltage levels in the staircase wave. In spite of this, the actual generation of an analogue quantity from digital data does in fact proceed in finite steps as a time delay must occur in clearing one number and setting the next. The inverse process on the other hand requires a finite time for execution as it is necessary to clear the counter and count pulses into it until the output of the amplifier coincides with the quantity to be converted. This time is quite small and is related to the precision of conversion.

Suppose we wish to convert some analogue quantity and maintain an accuracy of 0.1%. This would require that n = 10 and result in a staircase wave with 1024 steps. The conversion of the maximum amplitude would need approximately one thousand pulses. If a pulse frequency of (say) 500 Kc/s is used, then such a conversion would take approximately 2 milli secs. and the average time for converting a quantity varying between zero and a maximum with an accuracy of 1 in 1000 would be of the order of 1 milli-second.

The step by step nature of the conversion means that a smoothing or integrating process is needed in the output of the digital analogue devices and sampling of the input to an analogue digital converter must be carried out. Both these requirements place limits on the maximum rates of change of the data used in converting equipment.

4. Applications

The general field of application of digital analogue converters has been indicated already. Some particular examples will now be given in order to make clear the way in which these devices may be used in computing and control equipment. They are:-

(a) Digital analogue techniques for Electronic Digital Computers

(b) Digital store for electrical analogue computers

(c) Digital Elements in a Control System

(a) Analogue Techniques for Digital Computers

The fundamentally new approach to computing techniques that has followed the introduction of the automatic digital computer has resulted in some kind of bias towards the use of the practices already well established for analogue computers.

This bias, natural enough in the early stages of the development of a new method, is now beginning to disappear. The Manchester digital machine uses an analogue adder instead of one of the various forms of logical adder. As a matter of fact, in binary arithmetic there is little to choose between analogue and digital principles as the tolerance on the very few separate quantities that require physical representation may be large. An analogue adder requires that the digit "1" be represented by the amplitude of an appropriate signal (say) a pulse of amplitude 100 volts. Zero could be represented by no signal at all. Taking account of the "carry" signal the adder must combine three signals, each of which may be 0 or 100 volts, and give four voltage levels at the output (0, 100, 200, 300 volts) which would be used to discriminate the various combinations of resident and carry digits. It is clear that no specially accurate device would be required to achieve this.

Even when more voltage levels are produced as in the case of a staircase wave-form, the procedure described above permits the establishment of as many as 1000 voltage levels with a reliability normally expected in digital devices.

This fact has been made use of in the mechanism for selecting words in the memory of the Manchester machine. The electrostatic storage tube stores words of forty digits in single lines across the face of a 5" cathode ray tube. Up to 64 words may be written and stored in such a tube, one above the other and access to them is made by deflecting the reading/writing beam by means of the "Y" deflection system. The address of a given word is specified in the instruction and is written in decimal or binary form. If decimal notation is used then preliminary conversion to its binary equivalent is necessary before sending it on to the selecting mechanism. From then on the selecting process is simply a matter of direct digital analogue conversion. The given code establishes a voltage amplitude which when applied to the Y plates of the tube deflects the beam to the appropriate storage line.

(b) Digital Store for an Electrical Analogue Computer

A problem that hitherto has defied a completely satisfactory solution is that of providing functional data for an electrical analogue computer such as an electronic differential analyser. Methods already used include the winding of special potentiometers, a Fourier synthesiser, cathode ray tube mask and sundry other devices.

The employment of a digital store would provide a rather elegant solution to the problem especially from the point of view of versatility. Here again the direct conversion from digital to analogue would be required and this should present no unduly difficult problem. The whole unit would be rather elaborate when account is taken of the selecting mechanism and it would be necessary to make a careful study of the conversion rate and its relation to the maximum rate of charge of the functional data, bearing in mind that such data are usually functions of real time.

(c) <u>Digital elements of a Control System</u>

Mention has been made above of the many applications of mathematical computation to real time problems and their occurrence in control system. In addition to control systems that perform executive functions, there is a large class of instruments known as "simulators" which are half computers and half control systems. In all these the storage of digital data is of importance.

Before proceeding further it is worth noting the particular virtues of a digital store of the types recently developed for high speed computer work. They are:-

(i) Large capacity

(ii) High speed operation or small access time

(iii) Precision in representing data and as a corollary the ability of associated devices to manipulate the data without deterioration

The large capacity of a digital store is the property that may have the greatest influence on the use of digital methods in the control field. An obvious example is related to a class of machines known as profile or contouring machines. At present jigs and dies used for pressing automobile panels are operated from master cams which must be made by hand. The "three dimensional" cams used in service computing instruments are made by the laborious process of "stabbing" and hand finishing. A great advance could be made in this field if the co-ordinates of a surface were stored in a memory system and called forth and converted to analogue form to control the cutting head of the machine as it moves over the work.

On the score of their potentialities for high speed operation other control system problems arise. Many production processes in industry are now carried out at speeds in excess of the capacity of mechanical and relay counters. Although counting itself does not call for conversion mechanisms there are other applications where rapid access to stored data will introduce refinement to control processes.

The arbitrary precision that may be established with a digital standard cannot of course be maintained in the conversion process. In spite of this, however, there are slight advantages in being able to establish a reference in a universal form and quite often the dynamic mechanism linked directly with the process to be controlled may not suffer from "ageing" effects in the same way as say a voltage established by a battery or the setting of a bi-metallic strip.

The ability to retain the initial precision when manipulation of digital data is carried out is of great importance. Manipulation may refer of course to physical transfer of data or to its algebraic or arithmetical manipulation. The development of pulse code modulation in communication systems illustrates the possibilities resulting former interpretation. As mentioned earlier, control systems of sorts require units to carry out differentiation, integration and data smoothing and the advantages of carrying out these operations by digital means are very real. In all these applications it is usually necessary to carry out the direct and inverse conversions.

5. Conclusion

The summary given above of the principles of digital analogue conversion is not meant to be exhaustive. The principles only have been dealt with and it should be noted that a number of specialized devices (42) have already been developed for effecting conversions at higher speeds, greater accuracy and with less valves and standard circuit elements.

The remainder of this paper which seeks to show the possibilities for using these devices to extend the scope and usefulness of automatic computating machines has not been presented with any special desire to be realistic. The paper tends to probe a little into the future especially in view of the fact that the two major types of computing equipment have had to date a completely separate background and history. It will be surprising indeed if mating of the digital and analogue techniques does not produce some worthwhile progeny. However, its exact characteristics will remain for sometime yet a matter for conjecture.

Apart from these general remarks this brief survey seems to indicate that there is greater scope for conversion from digital to analogue form than is the case for the inverse process. This is fortunate for the former process is the more simple and in addition a conversion may be carried out in just the time necessary to establish the digital number in the counter register. If this time is still too great, a ring of registers may be used and the conversion time will only be delayed by the switching time needed to connect the summing amplifier to an adjacent storage register. After any one register has been sampled, steps may be taken to clear it and reset it to the $(n + x)$, the value of the stored data where n is the value just sampled and x the number of registers in the ring.

XIII. An Analogue Computer to solve Polynomial Equations with Real Coefficients

by E. O. Willoughby, G. A. Rose and W. G. Forte

Introduction

This paper describes an instrument built at Adelaide University for solving polynomial equations up to the fourth degree. It uses magslip resolver units and commercially available auto-transformers (variacs) for its principal computing elements.

It is basic to the solution of this type of equation that equations of odd degree have at least one real root, and roots other than real roots for both odd and even order equations occur in complex conjugate pairs.

Consider the general polynomial equation:

$$p\,(z) = a_0 + a_1 z + a_2 z^2 + \ldots\ldots\ldots\, a_n z^n = 0$$

or more conveniently, if $z = re^{j\theta}$,

$$p\,(z) = x + jy = a_0 + a_1 re^{j\theta} + a_2 r^2 e^{2j\theta} + \ldots\ldots a_n r^n e^{nj\theta} = 0$$

If p(z) is displayed as z is scanned over all possible values, then the n zero values of p(z) indicate the n roots. The computer displays p(z) $\cos\omega t$ (where $\omega/2\pi$ is a fixed supply frequency, t is time) through a narrow time gate, open when $\cos\omega t = 1$. Hence, p(z) is displayed corresponding to the values of z at $t = 0,\ 2\pi/\omega,\ 4\pi/\omega,\ \ldots\ldots$, as z is scanned in polar form. A typical display is shown in fig. 1.

The number of points defining the $p(z)$ path is equal to the scanning period divided by the supply period. The scanning period is the time taken for θ to change by 2π. The amplitude, r, is varied manually but must be held constant for periods of $2\pi/\omega$ so that z can assume all possible values. There is a minimum number of points necessary to define the $p(z)$ path. Hence the speed of operation is dependent directly on the arbitrary supply frequency.

2. Description of Instrument

The instrument consists of four computing units. They are:-

(a) The "$p(z) \cos \omega t$" generator

The function $p(z) \cos \omega t$ is produced by $(n + 1)$ identical resolvers operated as shown in table 1. The table indicates the circuit e.m.f.'s., assuming that the input variable $z = re^{j\theta}$ is fixed.

(b) The "r" unit

The voltages $a_n r^n \cos \omega t$, are supplied from n identical variacs which are ganged together. The input impedance of the variacs is raised by suitable parallel tuning capacitors. The variac connections are shown in fig. 2. The physical arrangement for varying r limits r to less than unity.

The computer operates in the range $0.1 < r < 1$. A simple transformation of the polynomial coefficients enables roots within the ranges

$$10^{k-1} < r < 10^{k} \qquad (k = 0, \pm 1, \pm 2 \ldots\ldots\ldots)$$

to be located.

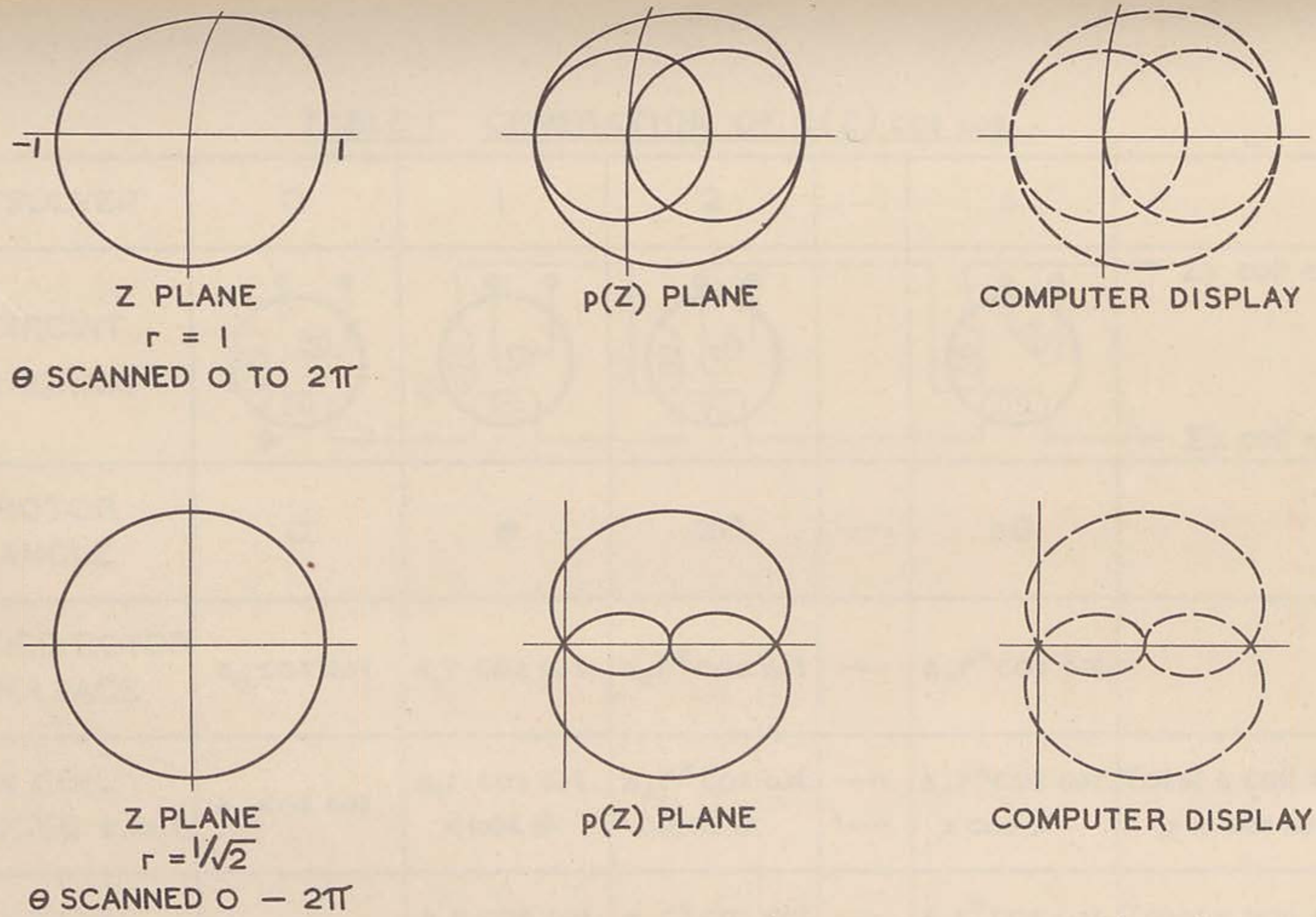
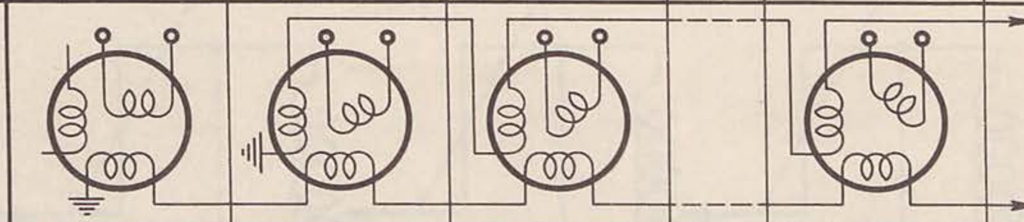


FIG.1. CATHODE RAY TUBE DISPLAY FOR SOLUTION OF $Z^3 - \frac{1}{2}Z + \frac{1}{2} = 0$.

REAL ROOT $Z = 1\angle\pi$.

COMPLEX ROOTS $Z = \frac{1}{2}\angle\pm\frac{\pi}{4}$.

TABLE I. GENERATION OF $\underline{p}(\underline{Z})\cos\omega t$.

RESOLVER	0	1	2	-----	n	
CIRCUIT DIAGRAM						Σy coil e.m.fs. Σx coil e.m.fs.
ROTOR ANGLE	0	θ	2θ	-----	$n\theta$	
APPLIED ROTOR VOLTAGE	$a_0 \cos \omega t$	$a_1 r \cos \omega t$	$a_2 r^2 \cos \omega t$	-----	$a_n r^n \cos \omega t$	
x COIL INDUCED e.m.f.	$a_0 \cos \omega t$	$a_1 r \cos \omega t$. $\times \cos\theta$	$a_2 r^2 \cos \omega t$. $\times \cos\theta$	----- -----	$a_n r^n \cos \omega t$. $\times \cos\theta$	Total x coil e.m.f. $= x \cos \omega t$.
y COIL INDUCED e.m.f.	0	$a_1 r \cos \omega t$. $\times \sin\theta$	$a_2 r^2 \cos \omega t$. $\times \sin\theta$	----- -----	$a_n r^n \cos \omega t$ $\times \sin\theta$	Total y coil e.m.f. $= y \cos \omega t$.

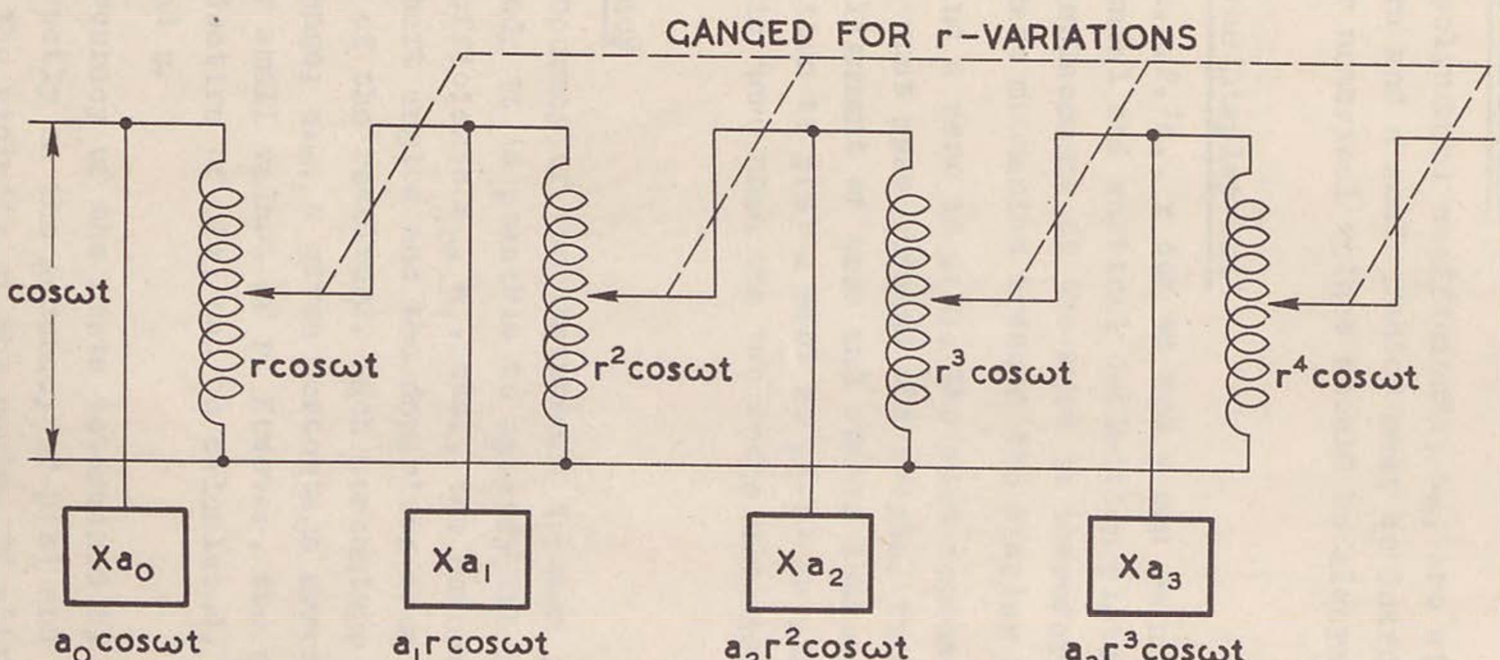


FIG. 2. METHOD OF INTERCONNECTION OF AUTO-TRANSFORMERS.

(c) Coefficients

The polynomial coefficients, a_n, are simulated by alternators and a scale factor must be introduced in order that their numerical values should be always less than unity.

(d) The Display Unit

The e.m.f.'s., x cos ωt and y cos ωt are applied to the horizontal and vertical deflection plates of a C.R.O. and the displacement of the spot is therefore p(z) cos ωt. The time gate mechanism reduces the display to p(z).

To find a zero in p(z), the spot representing the polynomial must pass through the origin. This requires accurate alignment of axes and stable tube mounting. However, if in addition to p(z) a spot at p(-z) is also displayed, then p(z) is zero when the two spots coincide.

3. Accuracy

The accuracy of the instrument depends upon the equation being solved. It is possible to specify the percentage error for the coefficients a_0, a_1, etc., the powers of r, the resolver shaft angles and the departure from sinusoidal resolution of the resolvers. Each percentage error varies over the range; e.g. a given percentage error in r^3 will be greater for small values of r. However, the overall error in the production of p(z) can be calculated, given the equation and z.

The accuracy of the roots determined by the computer depends directly on the accuracy of p(z) and the derivative of p(z) in the vicinity of the roots. If p'(z) = 0, then z may vary over a comparatively large range without producing a noticeable difference in p(z). Equations with multiple roots display this property. Hence a maximum error can be specified for the production of p(z), but it is not possible to specify the error in finding the roots. Inaccuracies due to small p' (z) are readily observed by the operator.

Discussion on papers XI, XII, XIII.

D. R. HARTREE - referred to a question on the use of the Pepinsky machine for double and triple Fourier summations, and said that not having used the machine he would refrain from comment. However, the installation of such a machine in England was at present under consideration, and one of the staff of the Cavendish Laboratory is in U.S.A. studying the machine. He commented also on the cinema integraph for solving integral equations and said that it had never been developed to a stage at which it was satisfactory in practice.

On the subject of digital-to-analogue conversions, he said that Mr. Blunden might have given the impression that the inverse conversion was of no practical importance. He pointed out that a machine known as MADDIDA, a differential analyser, has been produced commercially. It accepts analogue data and integrates digitally using a magnetic drum store. The output is recorded graphically.

He explained a method of converting shaft position into a true binary code by a simple switching arrangement. When a true binary code is used with such a mechanism there are certain steps (for example, changing from 3 to 4) where a number of digits change in going from one number to the next. This may lead to serious inaccuracy if these digits do not all change exactly together. A modified binary scale may be used to avoid this trouble:-

Decimal	Binary	Modified Binary
0	0000	0000
1	0001	0001
2	0010	0011
3	0011	0010
4	0100	0110
5	0101	0111

General Discussion

T. M. CHERRY - in opening the discussion, referred to the "Monte Carlo" methods and indicated that the random walk method of solving partial differential equations had been investigated by Dr. J. Todd, who had found that an accuracy of 2% to 3% could be obtained with two to three hundred walks. Professor Cherry also indicated that the random numbers were generated by the "middle of the square" method, i.e. by extracting the middle 8 digits from the square of an 8-digit number.

D. R. HARTREE - said that he was surprised at the accuracy described by Todd for as few as two to three hundred walks. It appeared too good.

He then referred to a question on round-off errors in a binary machine and said that he knew of no work that had been done on the subject. Huskey (59) has made a study of the accumulation of errors occurring in the integration of differential equations for which exact solutions were known and has shown that a departure from randomness in rounding errors may occur in some circumstances. Professor Hartree thought that the new machines themselves could be usefully applied to the study of the accumulation of the effects of rounding errors.

MISS H. N. TURNER - said that it was not clear to how whether sub-routines could be constructed which would be independent of the actual addresses of the storage locations referred to in the instructions of the sub-routine.

D. R. HARTREE - indicated that such sub-routines could be constructed by elaborating the programming. He illustrated the principles involved by generalising the simple program for the operation $|C(6)|$ to 4 (paper V, §3), and to a program for $|C(j)|$ to k. The sub-routine for the specific example is:-

	G K
m	A 6 F
m + 1	E 4 θ
m + 2	T 0 F
m + 3	S 0 F
m + 4	T 4 F

There are various ways in which this program might be generalised. For the EDSAC at Cambridge, this is done by an extension of the use of terminal code letters, as follows. The instructions are presented to the machine punched on the input tape in a form slightly different from that which they take in the store, the translation from one form to the other being carried out in the course of reading the tape; and in the course of this translation the values to be ascribed to j and k in any particular application of the sub-routine are incorporated in it. For the operation $|C(j)|$ to k, the instructions in addresses m and m + 4 have to be A j and T k; they are coded as A 0 H and T 0 N (punched as A H and T N, since a zero address does not have to be indicated explicitly on the tape), and the set of instructions beginning with A H is preceded by a set of tape entries specifying the interpretation to be given to the terminal code letters H and N in any particular application of this sub-routine. The set of instructions under which the machine is operating while reading instructions from the tape is such that a terminal code letter H results in C(45) being added to the instruction as punched, so that if in the course of reading the tape the number j (punched as P j F, see paper V, end of §2) has been placed in address 45, the instruction punched on the tape as A H becomes A j in the course of being read in, and this is the form in which it is placed in the store and used in the calculation; similarly terminal code letter N results in C(46) being added to the instruction as punched, and other terminal code letters are available. Thus the relation between tape entries and contents of storage locations is as follows:-

Address	Content	Tape Entry	Notes
		G K	
		J 45 K	✱
		P j F	H-parameter
		P k F	N-parameter
		T Z	∅
m	A j	A H	
m + 1	E (m + 4)	E 4 θ	
m + 2	T 0	T F	
m + 3	S 0	S F	
m + 4	T k	T N	

✱ Numbers (j, k) specified in following tape entries to be stored in addresses 45, 46.

∅ Next group of instructions to be placed in a set of consecutive storage positions beginning at m (determined by an earlier tape entry).

In such a simple example as this, this extended use of terminal code letters may seem a complication, but in large programs it results in a very substantial simplification of the process of programming.

For a fuller account of the ways in which values of parameters can be incorporated in a program for the EDSAC, see references (52) (57). An alternative way is by means of the auxiliary register known in the terminology of the Manchester machine, as the "B-register" (see paper V, end of §3).

R. A. SACK - found it hard to understand why the Gauss method of integration, using unequal intervals, should be preferable to Simpson's Rule. Automatic computers appear admirably suited for repetitive processes even at the expense of time.

D. R. HARTREE - said that the Gauss method had been worked out as a library sub-routine, hence no difficulty was experienced in incorporating it into a program. And for a given number of ordinates it gives a much higher accuracy than Simpson's rule.

Replying to a question on the number of high-speed automatic computers in existance to-day, he said there would be about thirty. Twenty of these are probably located in the United States and the remainder in Europe and Australia. Of these, about twelve could be regarded as completed machines.

In answering another question on the time taken to program a problem, Professor Hartree said that it was dependent on the library available and the experience of the people concerned with the programming. The program for the integration of a simple differential equation, using standard sub-routines, might be set up in a few hours, or less.

A. B. THOMAS - said that he was interested to hear that the most failures in the Cambridge machine occurred in the electro-mechanical units and in the passive elements of the electronic circuits. This showed that valves, commonly regarded as fragile and unreliable, were not the most likely source of failure; nevertheless, even greater reliability could be obtained by using special valves constructed for telephone repeaters. Mr. Thomas said also that the reliability of the electromechanical units would be improved by using cold cathode gas-filled tubes. The Post Office was introducing such tubes into new equipment in place of the electromechanical relays that have for so long dominated the field. There was also a possibility that gas-filled tubes may be useful in the electronic circuits.

D. R. HARTREE - in replying to Mr. Thomas said that the Cambridge group has examined the possibility of using gas-filled tubes in certain electronic circuits. The conclusion arrived at was that their speed of operation was somewhat too low and the extra reliability did not warrant the sacrifices in speed, expecially as future developments in computers will tend towards higher operating speeds.

Commenting on valve failures generally, Professor Hartree said that most valve failures occurred during the first few hundred hours of their life. Preliminary tests on new batches of valves would therefore remove the faulty ones. He also said that there seems to be considerable disagreement about factors influencing valve life, such as the effect of switching them on and off.

Referring to a question on automatic checking, Professor Hartree indicated that he had had no experience with machines with automatic checking facilities built into them. In general, he continued, most errors in a machine which operates on its own instructions are likely to produce results which are quite clearly nonsense. Any machine provided with an error correcting code might spend more time correcting itself than solving problems.

Professor Hartree concluded his remarks by referring to the electrolytic tank and mentioning the difficulties that a group at Cambridge had encountered in designing a suitable probe. He asked Mr. Hollway whether similar trouble had been met with locally.

D. L. HOLLWAY - said that trouble had been experienced originally from surface effects at the probes, and this led to measurements of impedance between the liquid and test probes made from different materials. Of these, copper was found to give the lowest and most reproducible impedance. As copper was not satisfactory from a mechanical point of view, beryllium copper was used instead. The amplifiers are arranged to avoid appreciable electrical loading of the probes; the potential probe is connected directly into an otherwise open circuited grid and the effective resistance facing the gradient probe is greater than 50 MΩ. The feed-back circuit ensures that both probes remain near earth potential. The error in the gradient measurement caused by the meniscus is minimised by setting the gradient amplifier gain before use by moving the probes into a test cell in which the gradient is known.

The uncertainty in gradient measurements is approximately $\pm$ 1% or $\pm$ $\frac{1}{2}$% when the probes are completely submerged. This is sufficiently accurate for tracing electron paths in beam deflection electron tubes but it is not sufficient to allow measurement of aberrations in electrostatic lens systems of long focal length and this may have been the class of problem investigated at Cambridge.

W. R. BLUNDEN - said that he had been impressed by the potentialities of the "Monte Carlo" processes. He thought the method seemed so powerful that special purpose machines, not necessarily digital ones, should be made to exploit the process. They would be extremely useful for solving certain partial differential equations of common occurrence in mathematical physics. He pointed out that considerable work had already been done in an allied field on high speed correlation of statistical data in prediction theory. As a matter of fact, the C.S.I.R.O. Differential Analyser had been used successfully to predict unwanted frequencies in a paper making machine by evaluating auto-correlation and spectral density coefficients from graphical records of the thickness of the paper made by the particular machine. This work is normally not suitable for a mechanical differential analyser as it is too slow.

Exhibition of Equipment and Demonstrations

During the conference there was a display of computing equipment in the Drawing Office of the Electrical Engineering Department and in the C.S.I.R.O. Radiophysics Laboratory. Much of the equipment was loaned by various manufacturers and distributors of commercial type calculating machines. The following were included in the demonstration:-

At the University

1. The C.S.I.R.O. Differential Analyser by C.S.I.R.O. Mathematical Instruments Section
2. National, Type 3000 - Specially modified and exhibited by C.S.I.R.O. Radiophysics Laboratory
3. Typical new model multi-register machine by National Cash Register Co.
4. Type 2000, multi-register machine by National Cash Register Co.
5. Class M 200 6 register and a 10 register machine by Burroughs.
6. "Sunstrand" Early Model D by Stott and Underwood. Messrs. Stott and Underwood are exhibiting this model by special request as it is a "vintage" machine and of special historical interest.
7. Various hand machines by McDougalls
8. I.B.M. 602 A Calculating Punch - Multi-counter Multi-register Plug board sequenced Machine by I.B.M.
9. Multiplying Punch
   Tabulator
   Full Counting Sorter
   Universal Automatic Key Punch by Kalamazoo (Aust.) Ltd.

At the Radiophysics Laboratory

10. C.S.I.R.O. Mark I Electronic Computer by C.S.I.R.O. Radiophysics Laboratory

11. Hollerith Punch Card Machine by C.S.I.R.O. Radiophysics Laboratory

Acknowledgments

The organizing committee, on behalf of those attending the conference, expresses its appreciation of the initiative taken by the Executive of the Commonwealth Scientific and Industrial Research Organization in arranging the conference. The committee is indebted to the University of Sydney for making available suitable accommodation for the meetings, and to Sydney University Union for providing facilities for meals.

The thanks of the committee are due also to the representatives of business machine interests and to officers of C.S.I.R.O. for setting up and operating equipment for purposes of demonstration.

The major responsibility for the conference fell on the shoulders of the organizing secretary, Mr. W. R. Blunden, ably assisted by Miss D. B. Power, Miss H. J. Cole, Miss M. A. Adamson, Mr. M. W. Allen and Mr. J. E. Todd, of C.S.I.R.O.

The Committee is particularly grateful to Professor Hartree, who, in spite of illness contracted during his visit to Australia, contributed unsparingly to the discussions, and whose presence assured the success of the conference.

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